Principia of Global Trade: Measuring Trade Mass in Global Networks with a Self-Consistent Gravity Model

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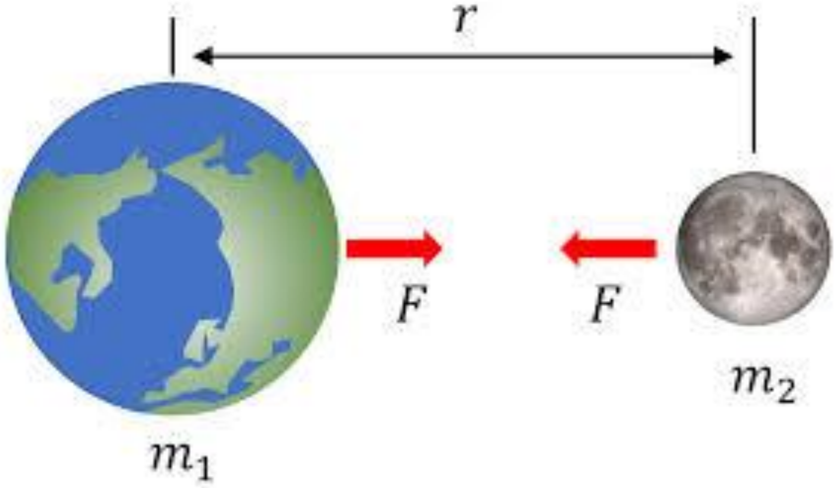
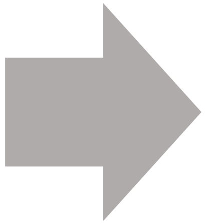
Gravity model of trade

Law of universal gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

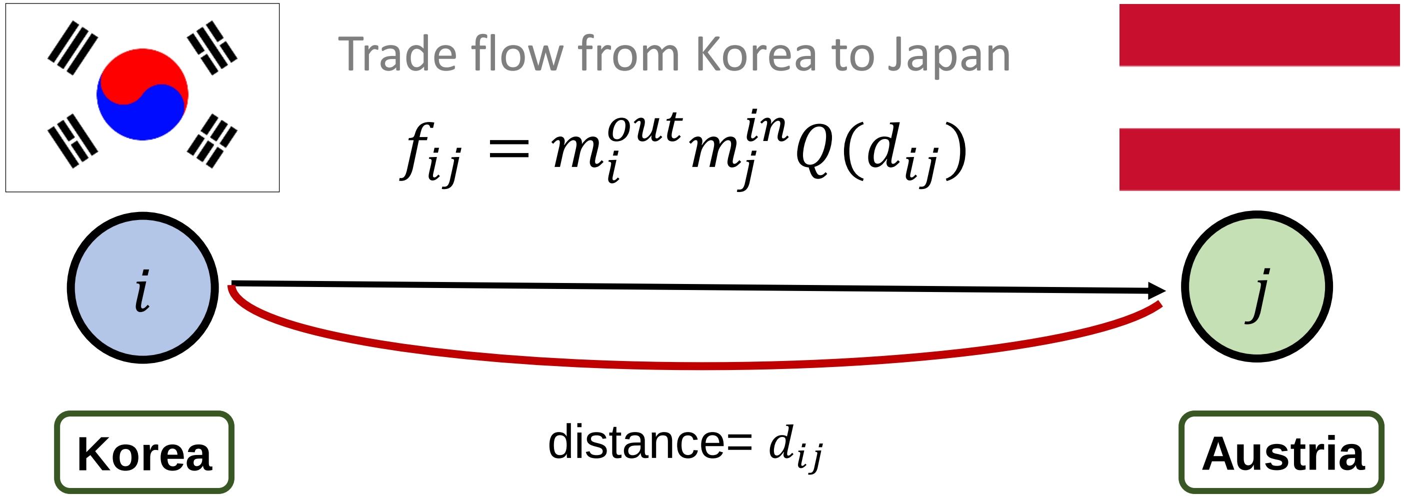




Gravity model of trade

- Newton's gravity: Objects with mass attract each other
- Inference of mass (m) and spatial dependency ($1/r^2$) from the observation of celestial mechanics (F)
- Can a similar principle be applied to global trade?

Gravity model of trade

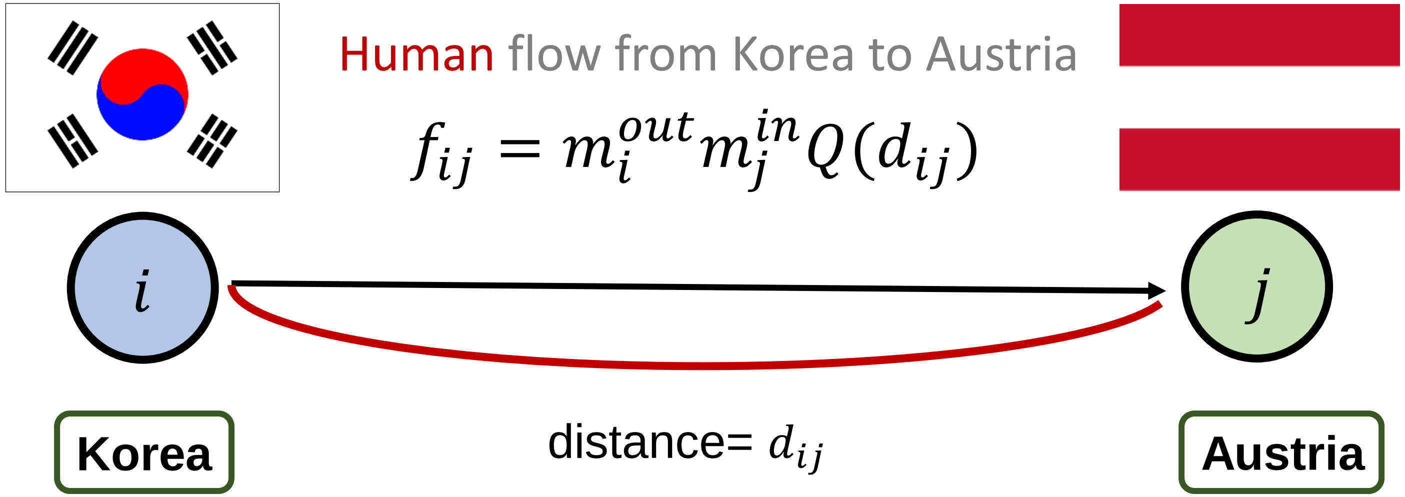


$m_i^{out} = capacity\ for\ export$

$m_j^{in} = capacity\ for\ import$

- Gravity model: Theoretical model for patterns of international trade
- Trade flow(f_{ij}) depends on the mass of exporting country (m_i^{out}) and importing country (m_j^{in})
- It also depends on their distance (r_{ij}) through deterrence function $Q(r_{ij})$

Gravity model of migration



$$m_i^{out} = capacity\ for\ outward\ flow$$

$$m_j^{in} = capacity\ for\ inward\ flow$$

- Traffic flow(f_{ij}) also can be expressed with outward mass (m_i^{out}) and inward mass (m_j^{in})

Gravity model of trade

Known

Unknown

$$f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$$

Unknown

- Dilemma of gravity model : Mass and Deterrence function cannot be derived simultaneously

Gravity model of trade

Known

Unknown

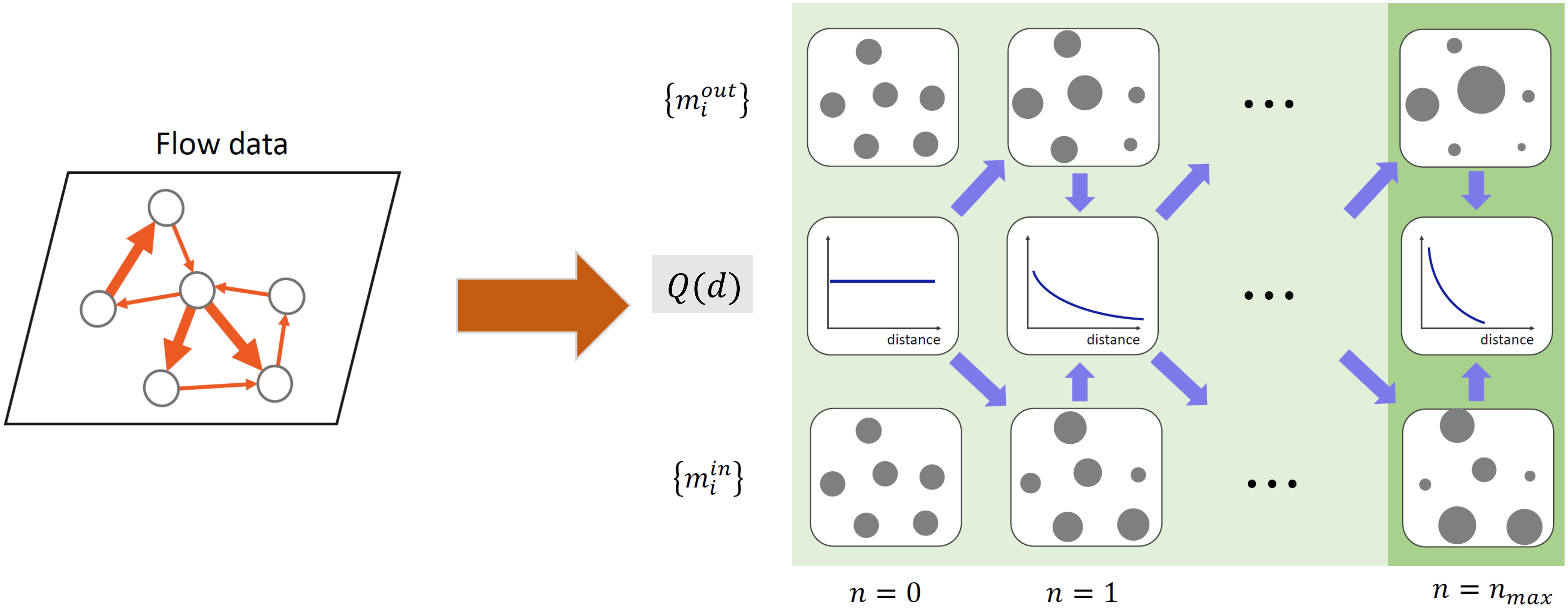
$$\boxed{f_{ij}} = \boxed{m_i^{out} m_j^{in}} \boxed{Q(d_{ij})} \quad \Rightarrow \quad Q(d_{ij}) = \left\langle \frac{f_{ij}}{m_i^{out} m_j^{in}} \right\rangle$$

Unknown → approximated

Ex 1) population (migration)
Ex 2) GDP (trade)
Ex 3) total export, import

- Dilemma of gravity model : Mass and Deterrence function cannot be derived simultaneously
- Conventional method: Masses are approximated by proxy attribute (GDP, population, etc)
- The need for self-consistent inference of quantities without proxies arises.

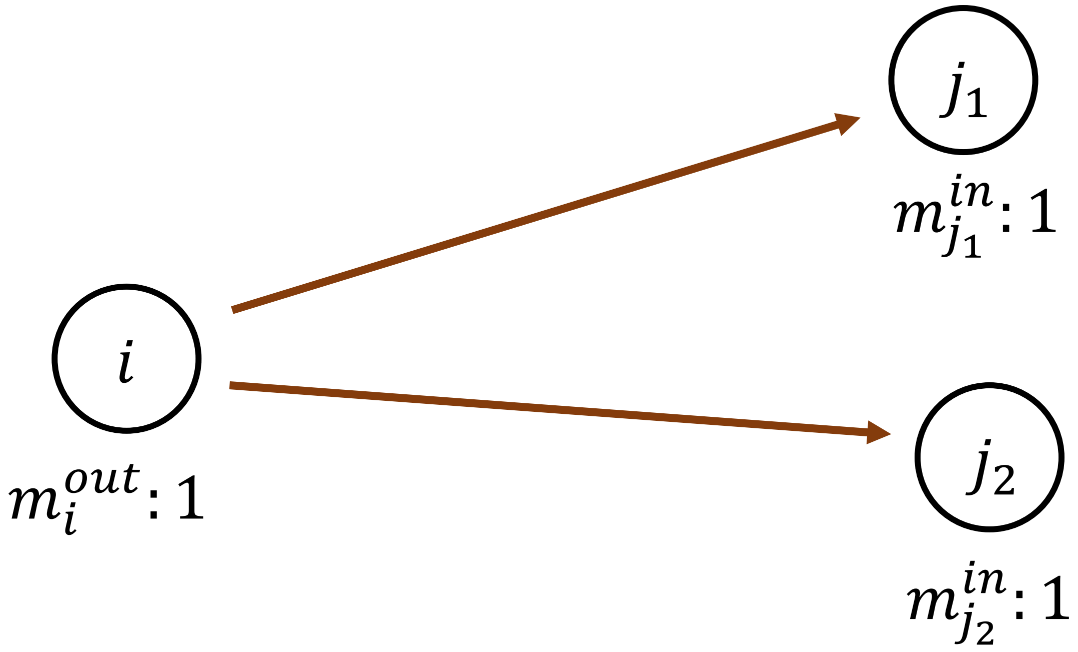
Self-consistent inference formulation



- Idea: Initialize mass and Q with simple distribution and sequentially update each other
- Mass is updated with temporal Q, and Q is updated with temporal mass distribution
- Update until equilibrium for effective data representation

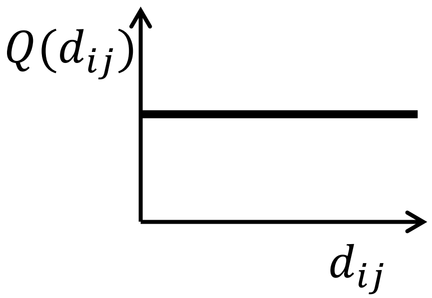
Self-consistent inference formulation

- Initialization



- $f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$

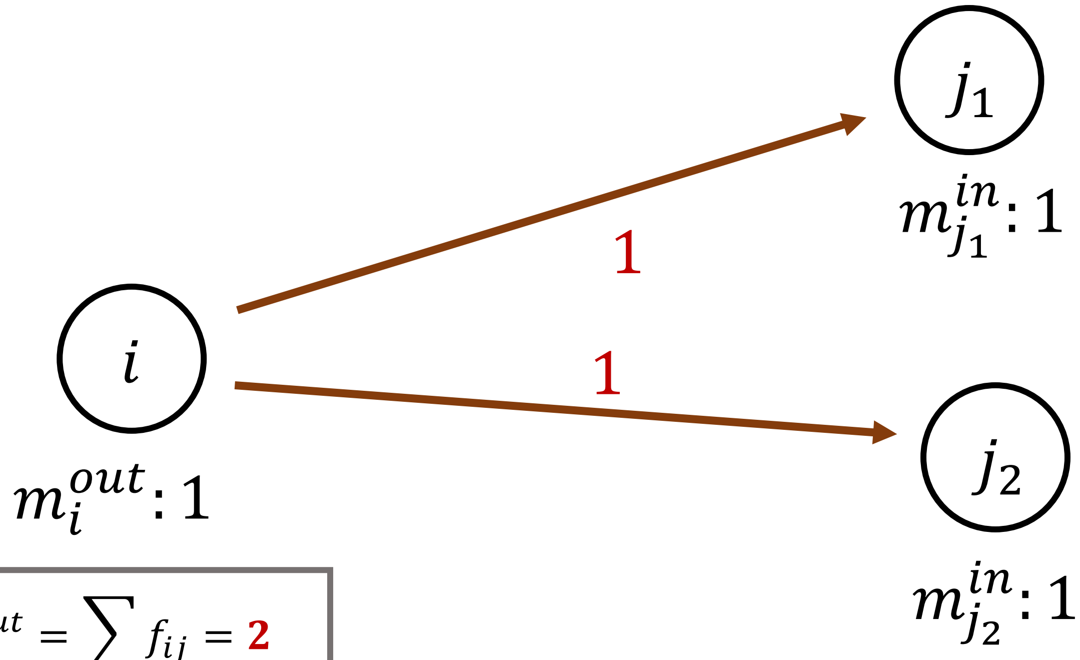
$$Q(d_{ij}) = 1$$



- Find appropriate mass and deterrence function for the network
- Each quantity is initialized as a uniform distribution

Self-consistent inference formulation

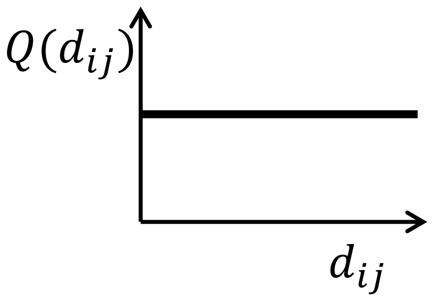
- Temporal estimation



$$\boxed{S_i^{out} = \sum_j f_{ij} = \mathbf{2}}$$

- $f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$

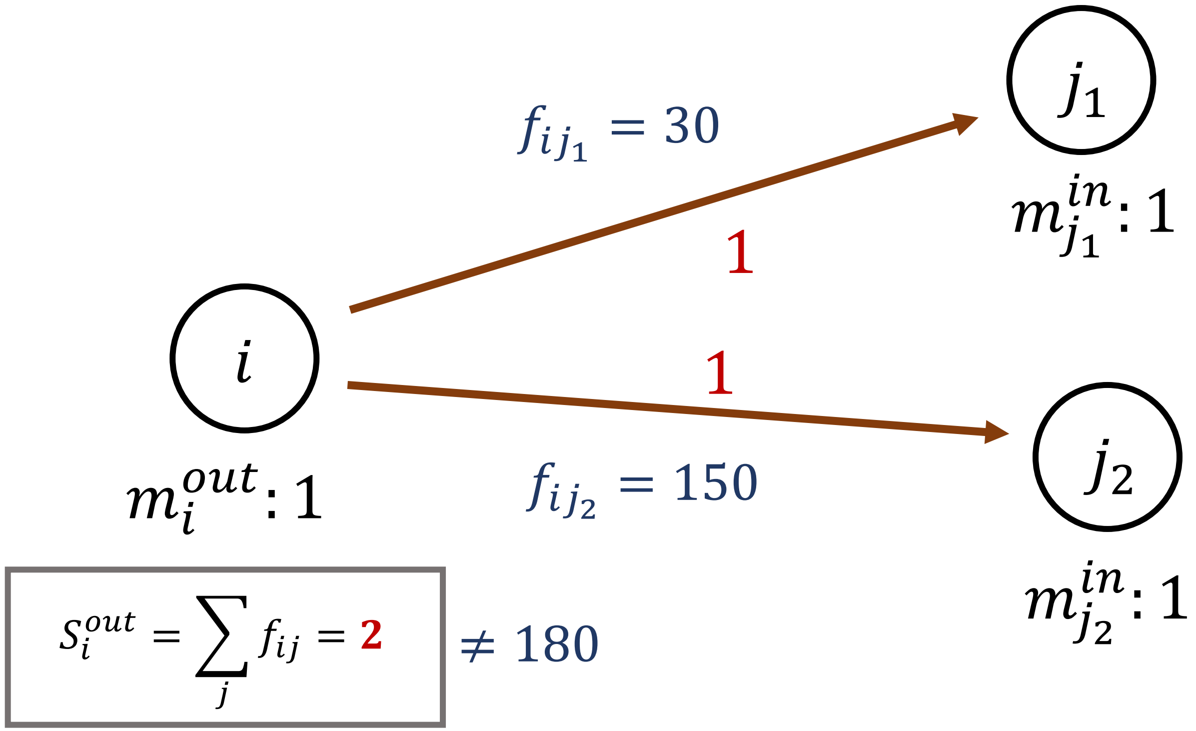
$$Q(d_{ij}) = 1$$



- Find appropriate mass and deterrence function for the network
- Each quantity is initialized as a uniform distribution
- Calculate a temporal estimation of total flow (S_i^{out})

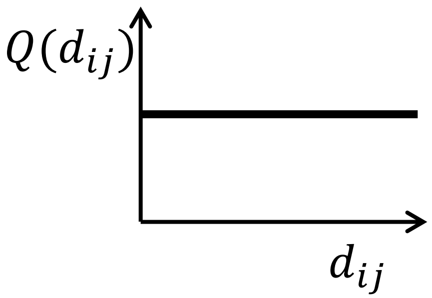
Self-consistent inference formulation

- Comparison with data



- $f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$

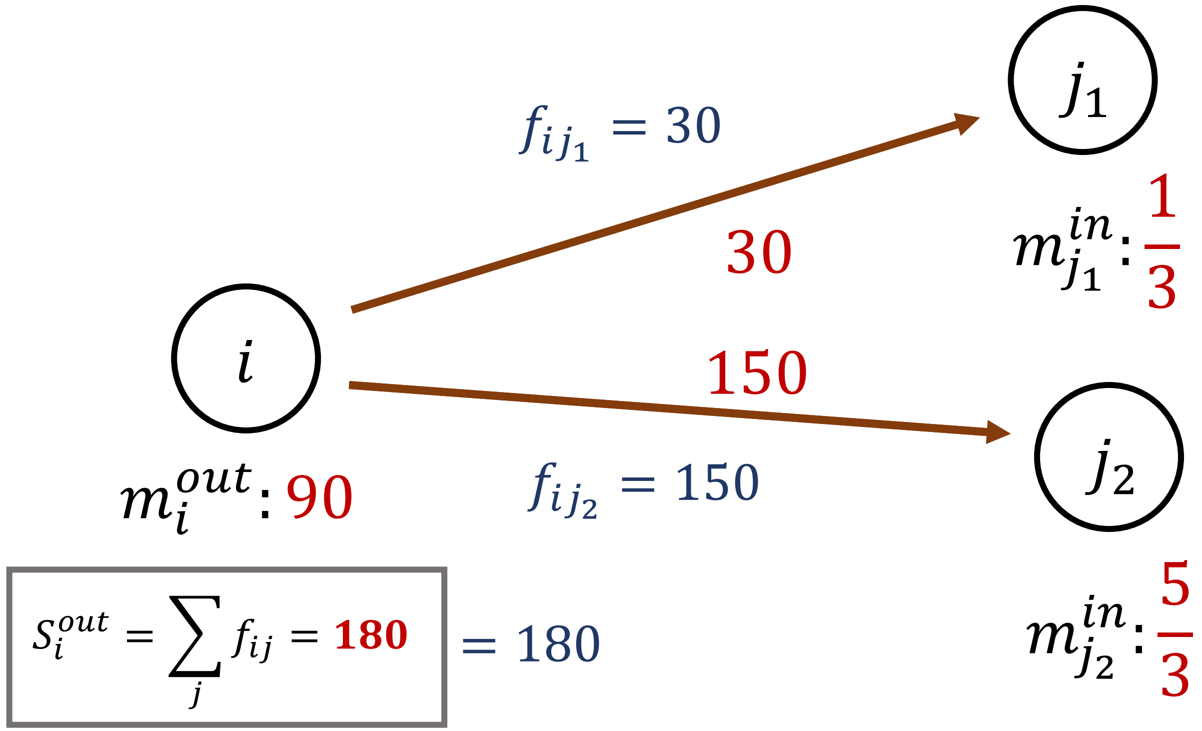
$$Q(d_{ij}) = 1$$



- Compare with the total flow in the real data
- Adjust estimation of masses to coincide with the real data

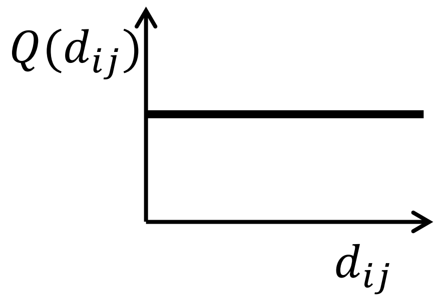
Self-consistent inference formulation

- Update masses



- $f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$

$$Q(d_{ij}) = 1$$

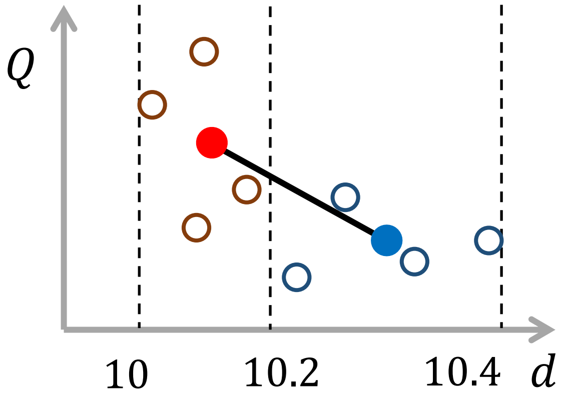


- Compare with the total flow in the real data
- Adjust estimation of masses to coincide with the real data

Numerical inference in gravity model

- Update deterrence function

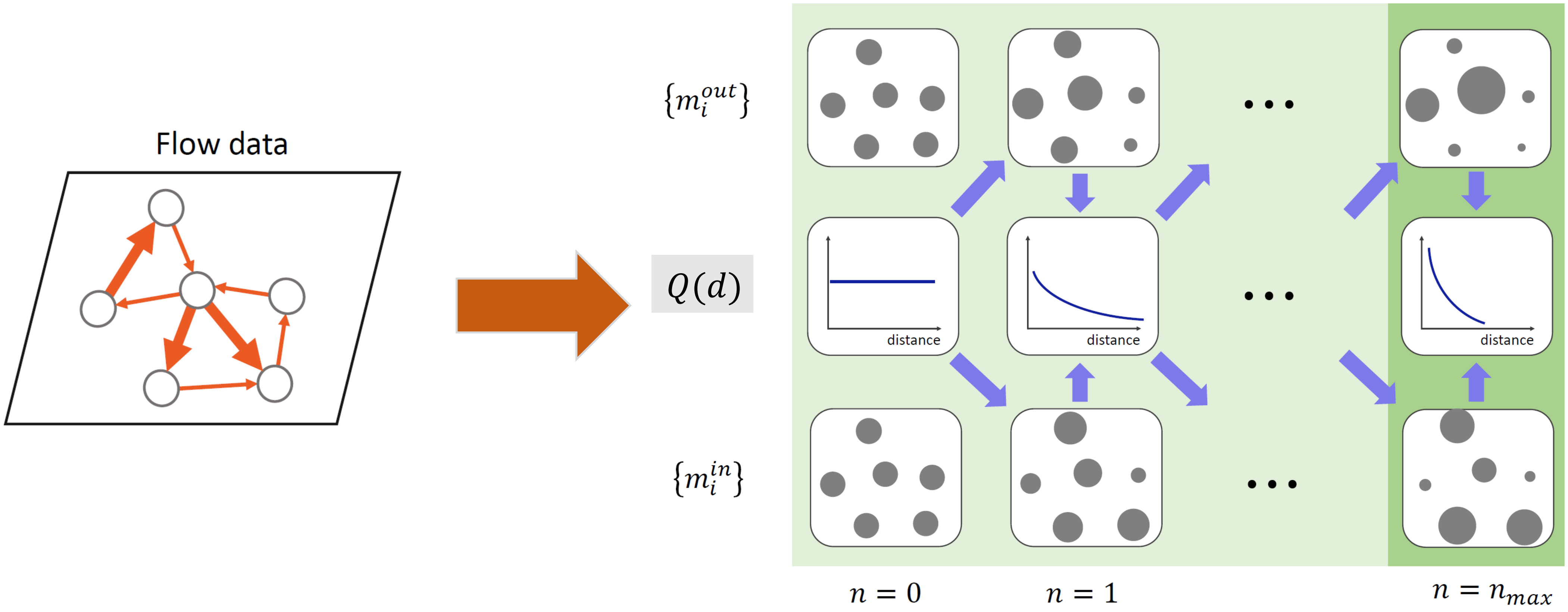
$$f_{ij} = \boxed{m_i^{out} m_j^{in}} Q(d_{ij}) \Rightarrow Q_{next}(d_{ij}) = \left\langle \frac{f_{ij}}{m_i^{out} m_j^{in}} \right\rangle_{d_{ij}}$$



$$\Rightarrow Q_{next}(10) = \left\langle \frac{f_{ij}}{m_i^{out} m_j^{in}} \right\rangle_{d_{ij=10}}$$

- Make a new deterrence function (Q_next) with new estimation of masses (m_i^{out}, m_j^{in}) and data (f_{ij})
- The average of the corresponding pairs determines each point
- Piecewise linear approximation

Gravity model of trade

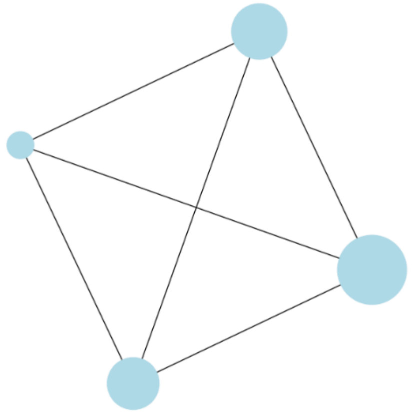


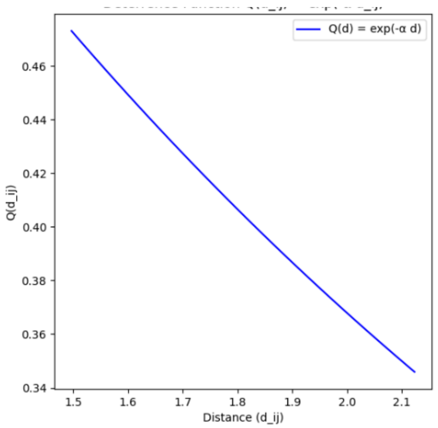
- Initialize each quantity as uniform distribution
- Iteratively update the mass and deterrence function with each other
- Will it converge to the ground truth of system?

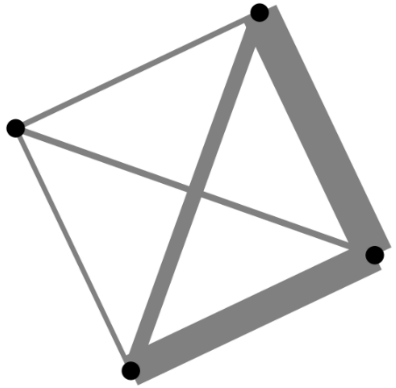
Verification with synthetic data

Network

$$m_i^{in}, m_i^{out} \qquad Q(d_{ij}) \qquad f_{ij} = m_i^{out} m_j^{in} Q(d_{ij})$$



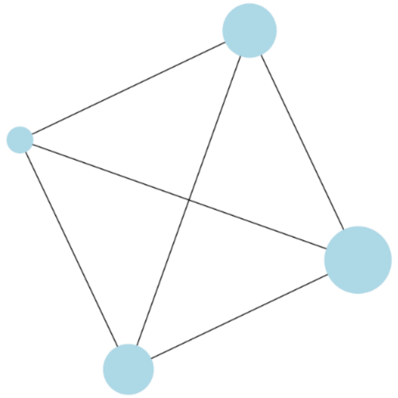


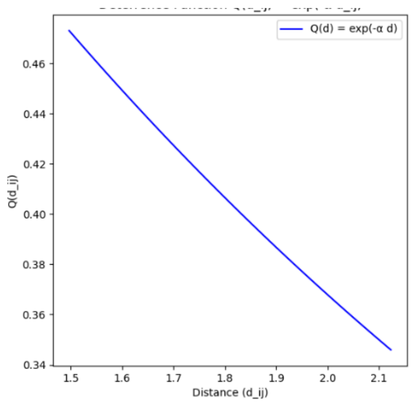


- Make a synthetic network to verify the algorithm
- Topology of international trade network
- Arbitrary mass distribution, deterrence function and flow distribution

Verification with synthetic data

Result





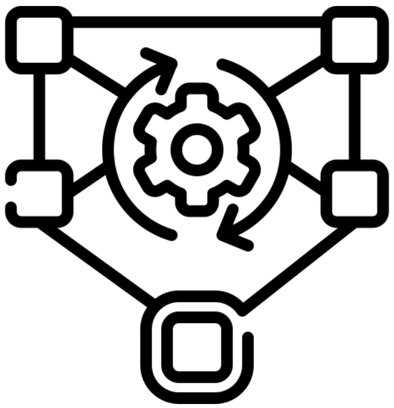
① Model

$\widetilde{m}_i^{in}, \widetilde{m}_i^{out}$

$\tilde{Q}(d_{ij})$

f_{ij}

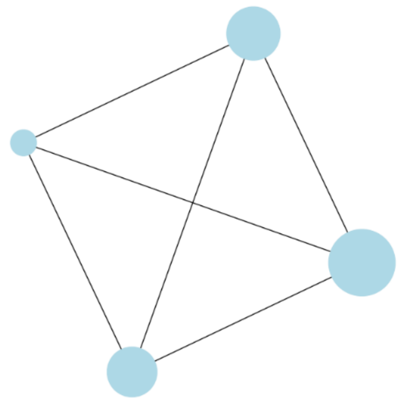
② Algorithm

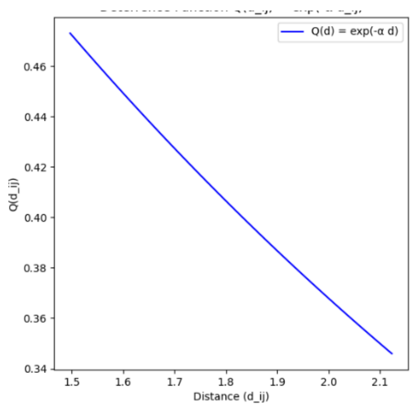


③ Inference

$\overline{m}_i^{in}, \overline{m}_i^{out}$

$\bar{Q}(d_{ij})$



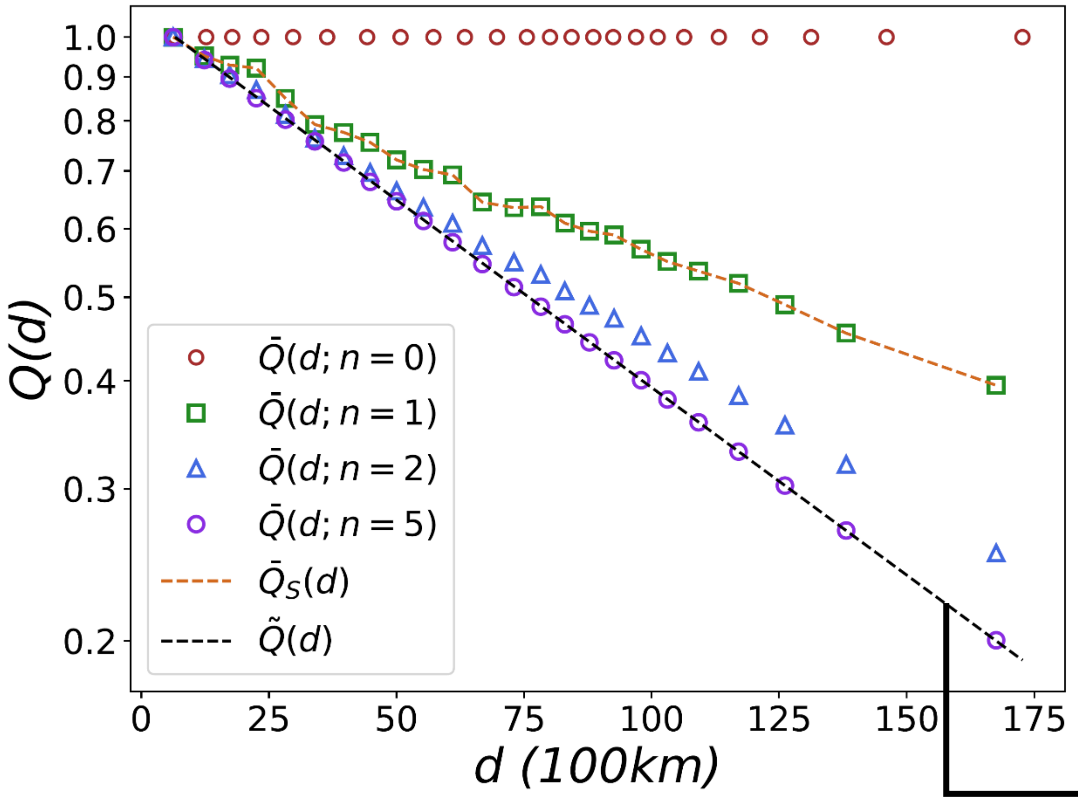


④ Evaluation

- Inference of masses and deterrence function with f_{ij}
- Comparison between real and inferred quantities

Model verification with synthetic data

- **Deterrence function**



$$Q^S(d_{ij}) = \left\langle \frac{f_{ij}}{S_i^{out} S_j^{in}} \right\rangle_{d_{ij}}$$

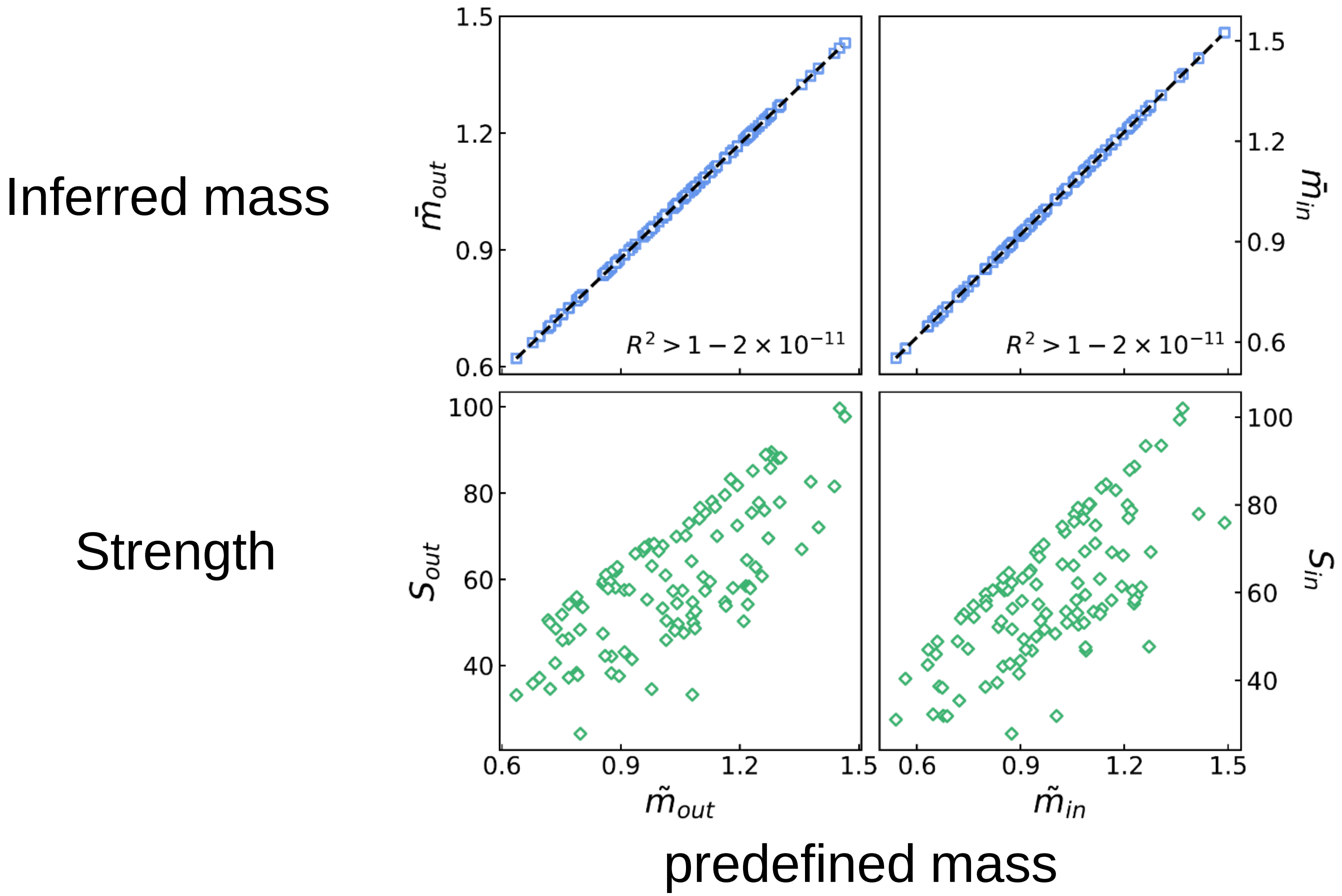
$$Q_{next}(d_{ij}) = \left\langle \frac{f_{ij}}{m_i^{out} m_j^{in}} \right\rangle_{d_{ij}}$$

Ground truth

- Inferred deterrence function is well matched with predefined shape.
- Strength approximation produce a wrong result

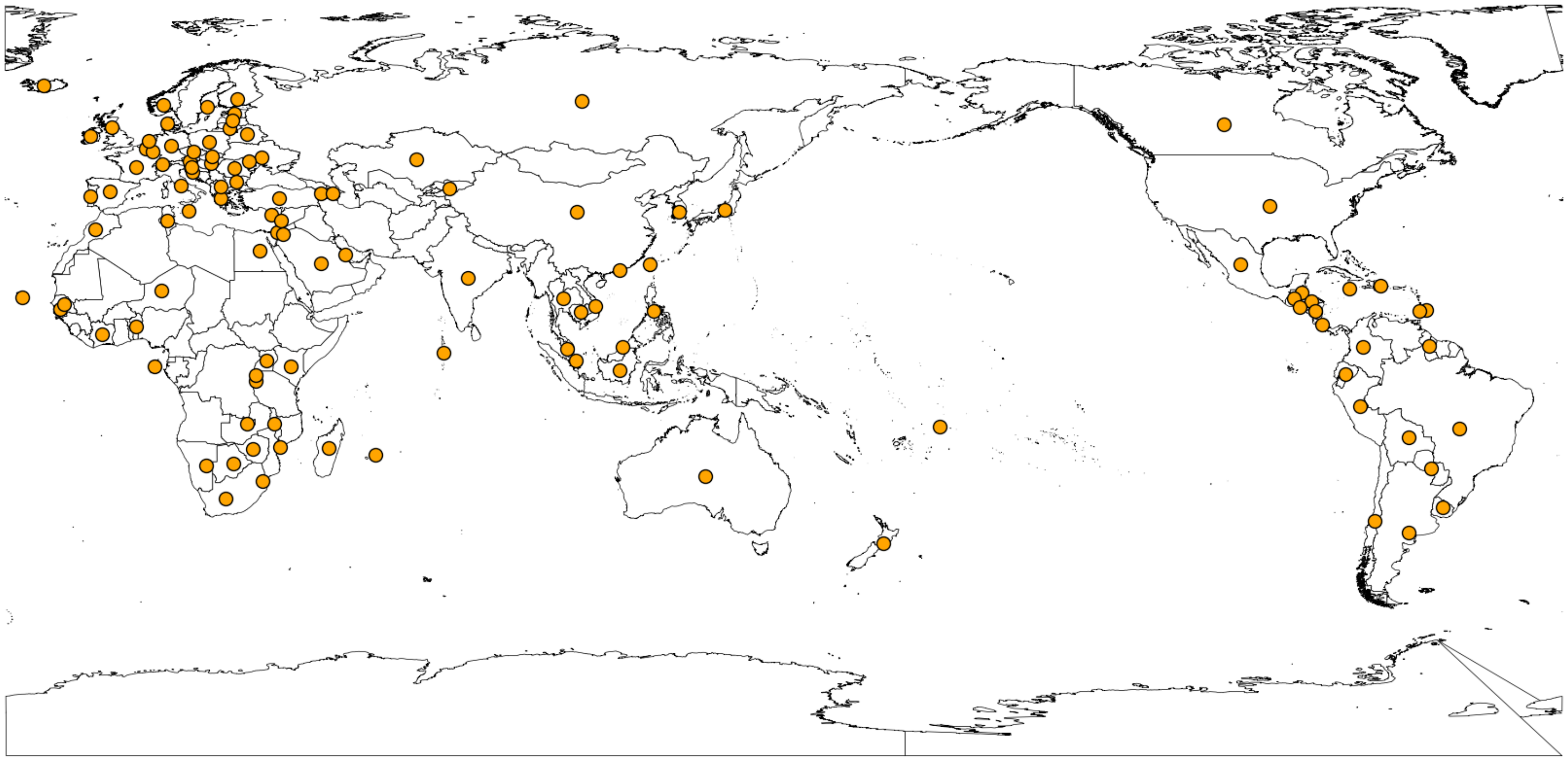
Model verification with synthetic data

▪ Mass distribution



- Inferred mass distribution is accord with predefined distribution.
- Strength approximation produces a wrong result

Model verification with real network

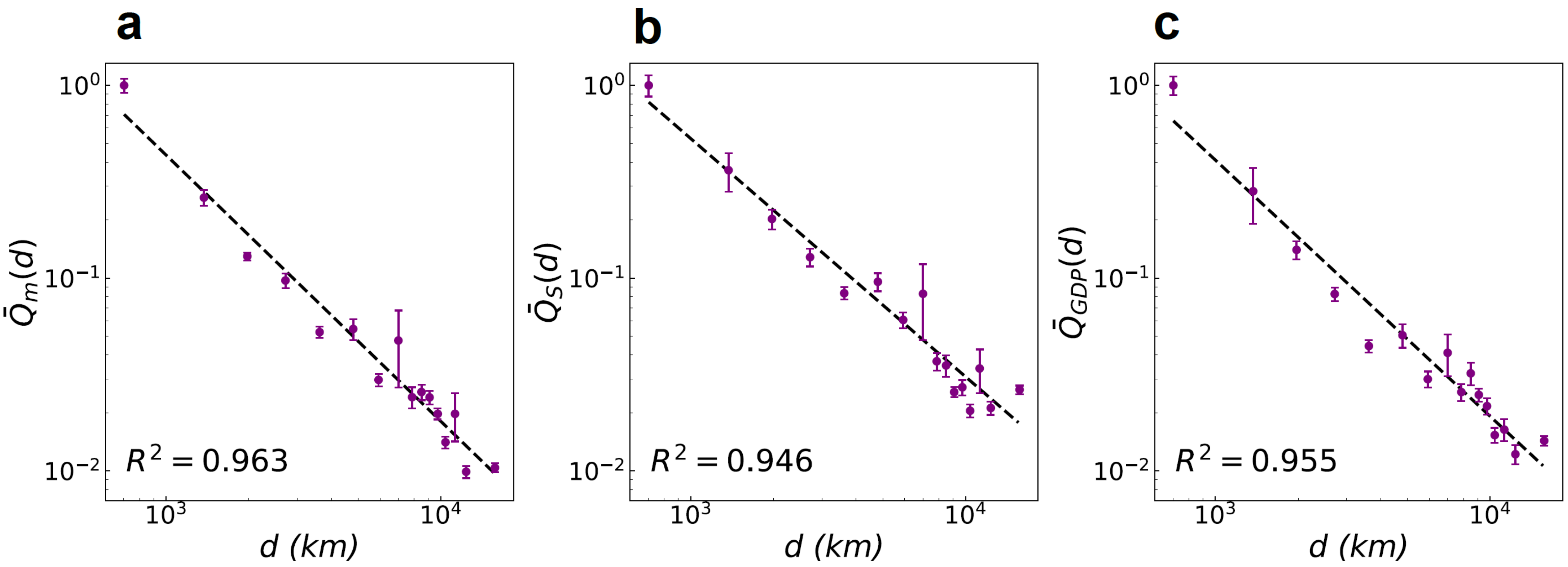


- International trade network: example to see the global economic trend
- 221 nodes, weight = total trade volume in 2021
- Distance between countries : geodesic

Model verification with real data

Result

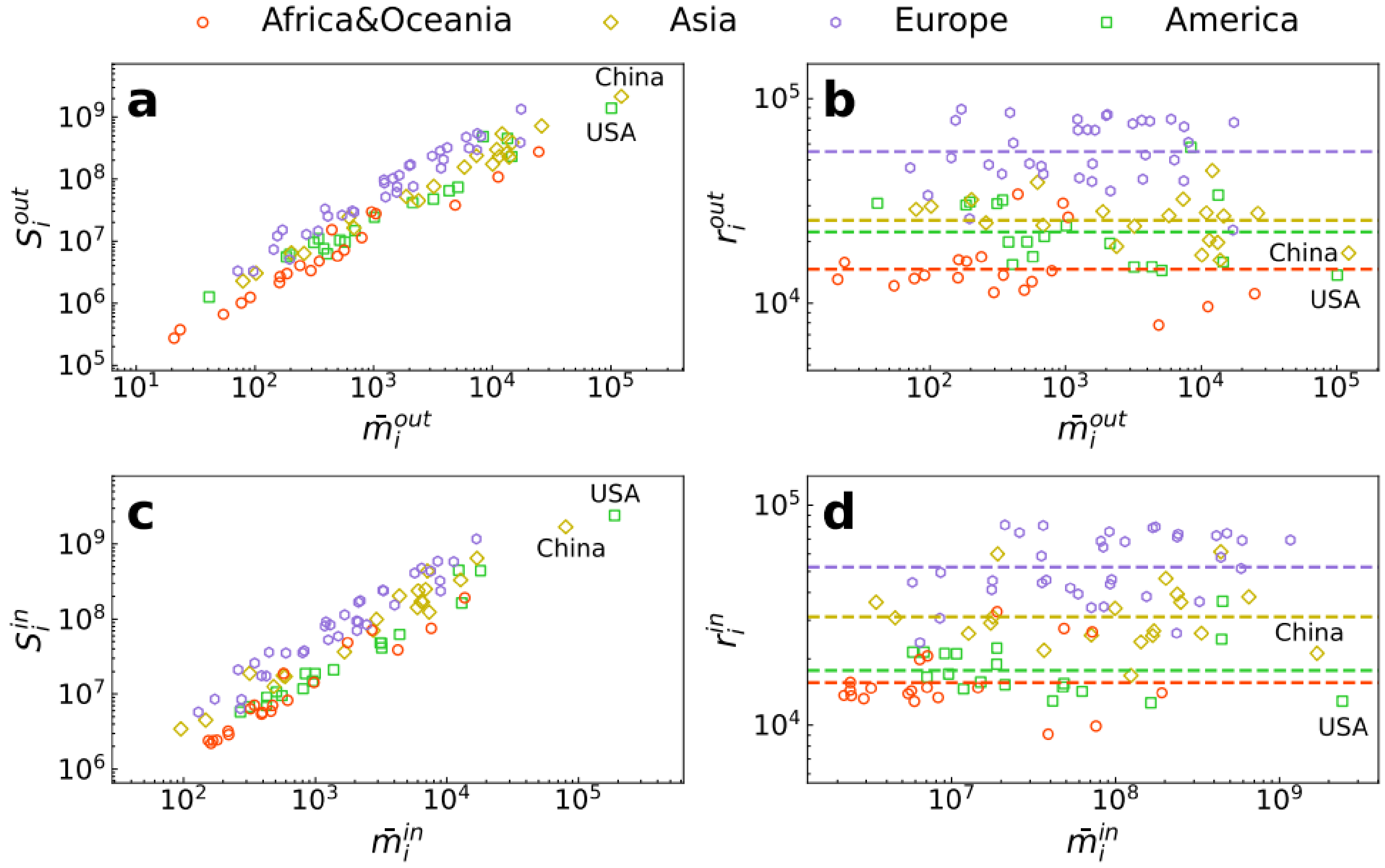
- **Deterrence function**



- Looks like power-law
- Our model shows smaller error bars and consistent form

Model verification with real data

- **Mass vs Strength**

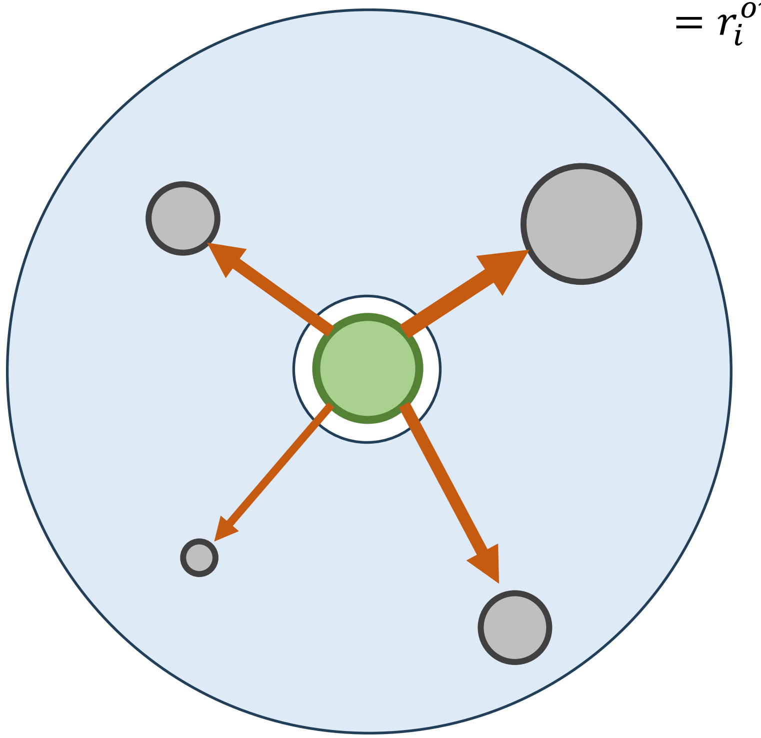


Total export

Export mass

$$S_i^{out} = \sum_j f_{ij} = \boxed{m_i^{out}} \boxed{\sum_j m_j^{in} Q(d_{ij})}$$

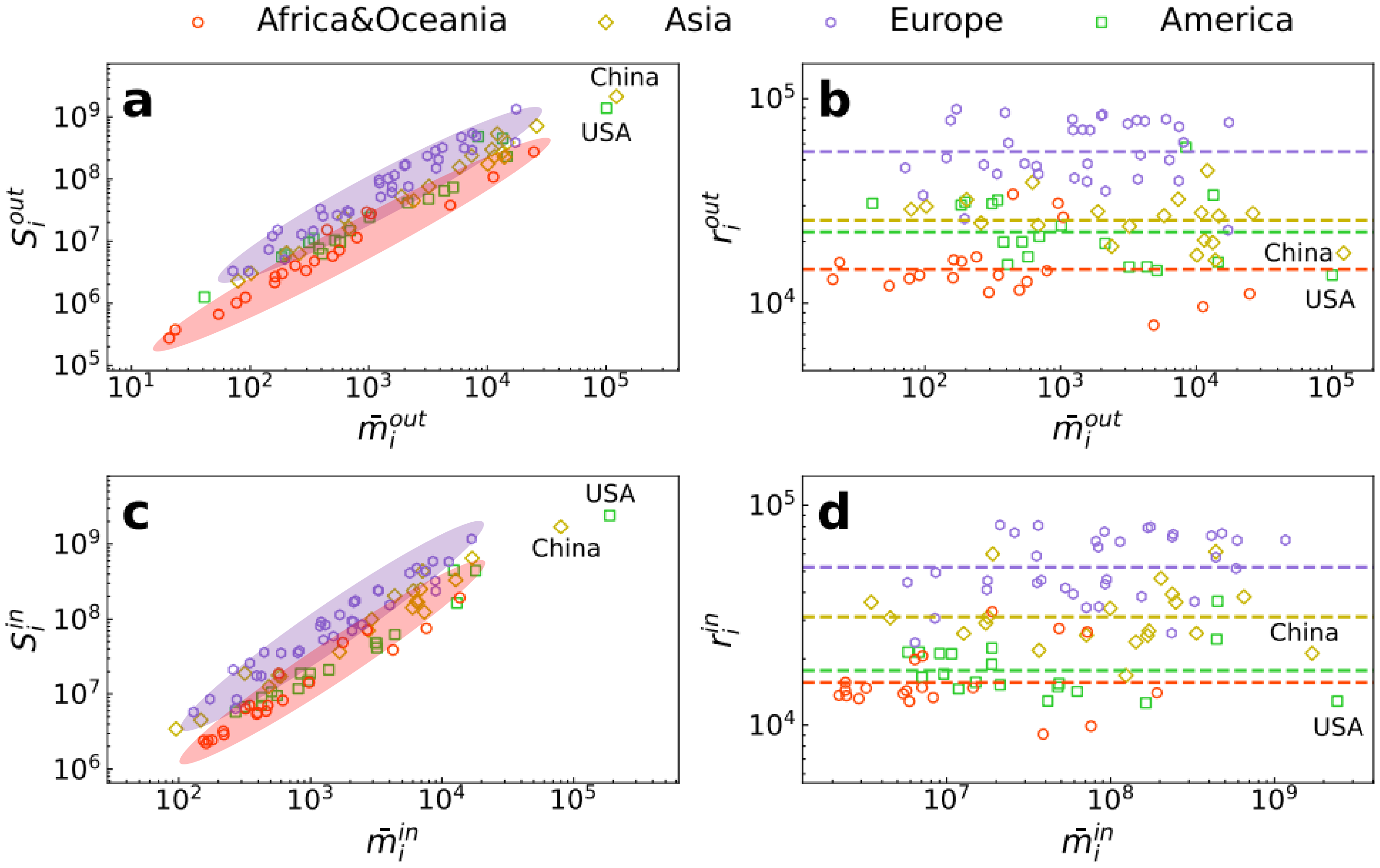
Advantage Index $= r_i^{out}$



- Comparison between strength and mass of each node
- The ratio between them expresses their surrounding economic environment?

Model verification with real data

▪ Mass vs Strength

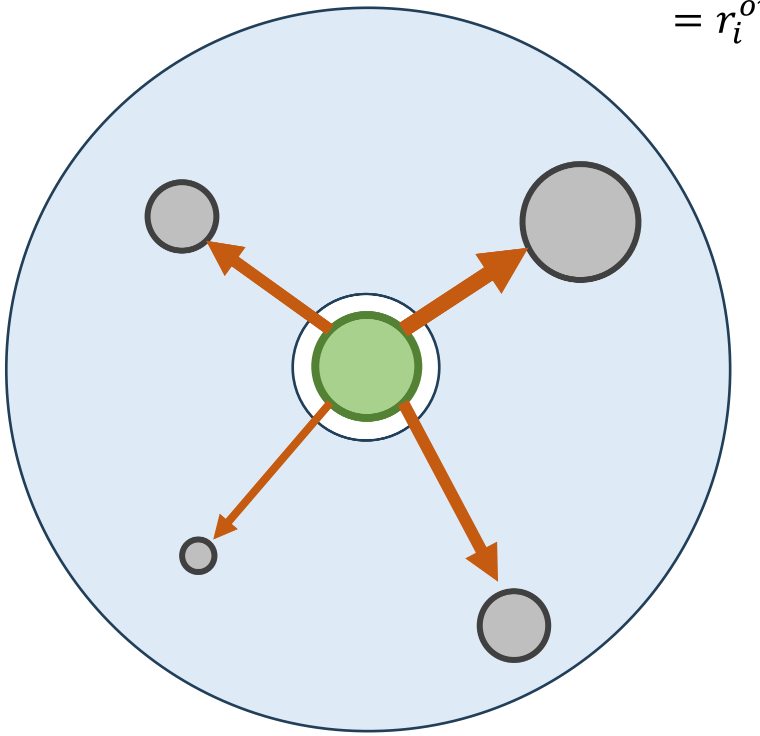


Total export

Export mass

$$S_i^{out} = \sum_j f_{ij} = \boxed{m_i^{out}} \boxed{\sum_j m_j^{in} Q(d_{ij})}$$

Advantage Index $= r_i^{out}$

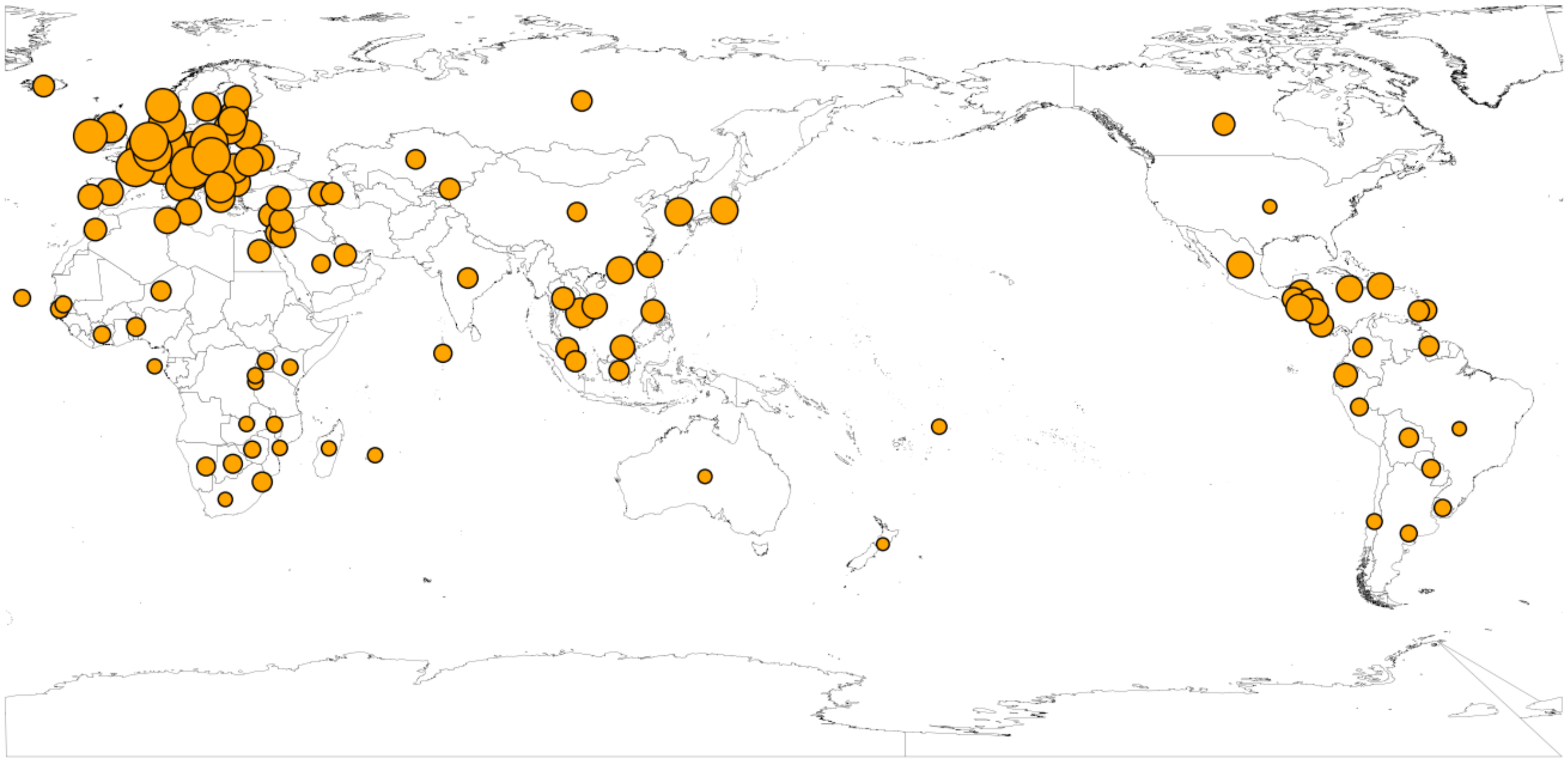


- Comparison between strength and mass of each node
- Strength-mass ratio reveals each country's market effect

Model verification with real data

Result

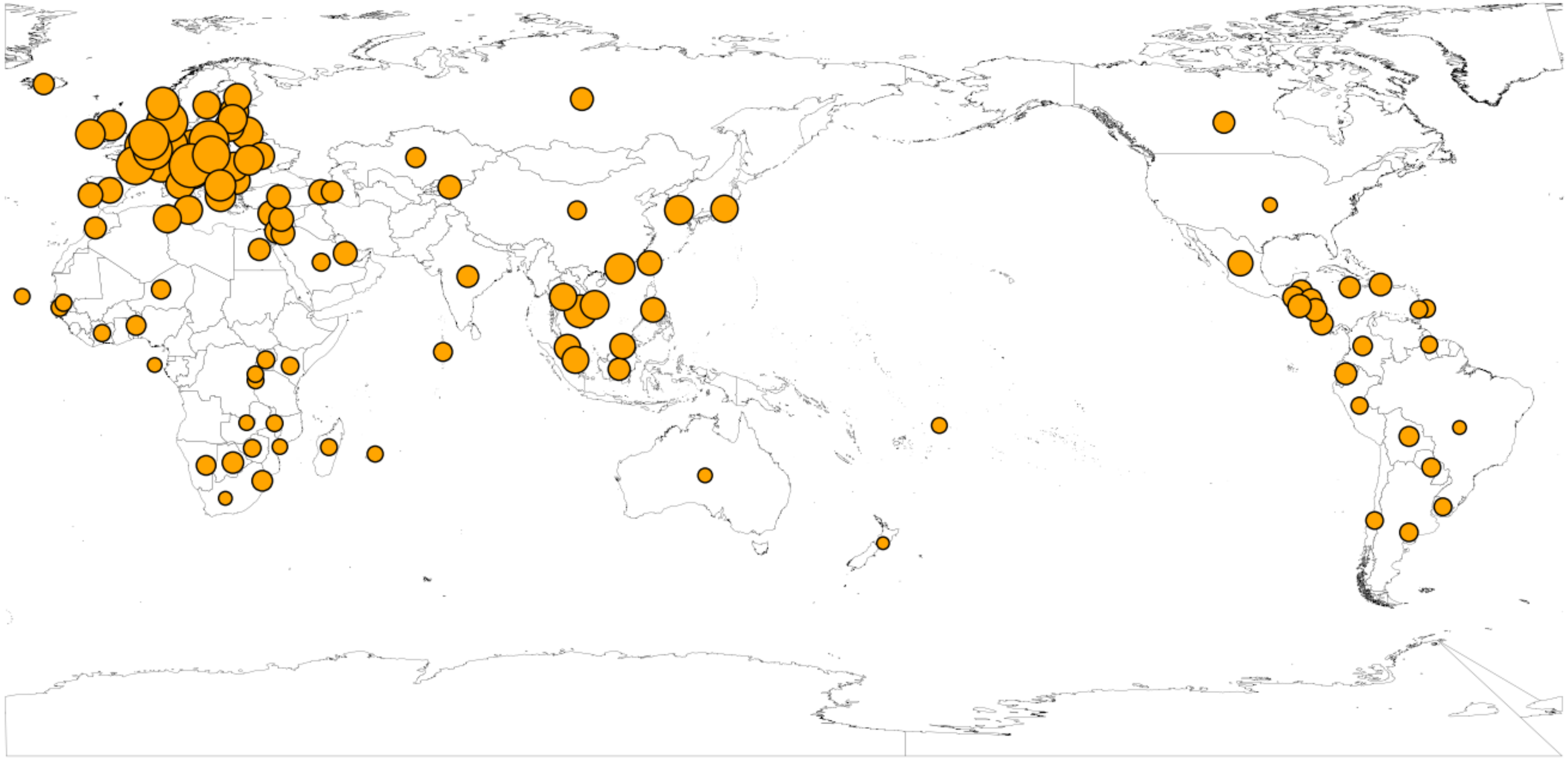
$$r_i^{out} = S_i^{out} / m_i^{out}$$



- Advantage index distribution for export
- European countries reveal the highest market effect

Model verification with real data

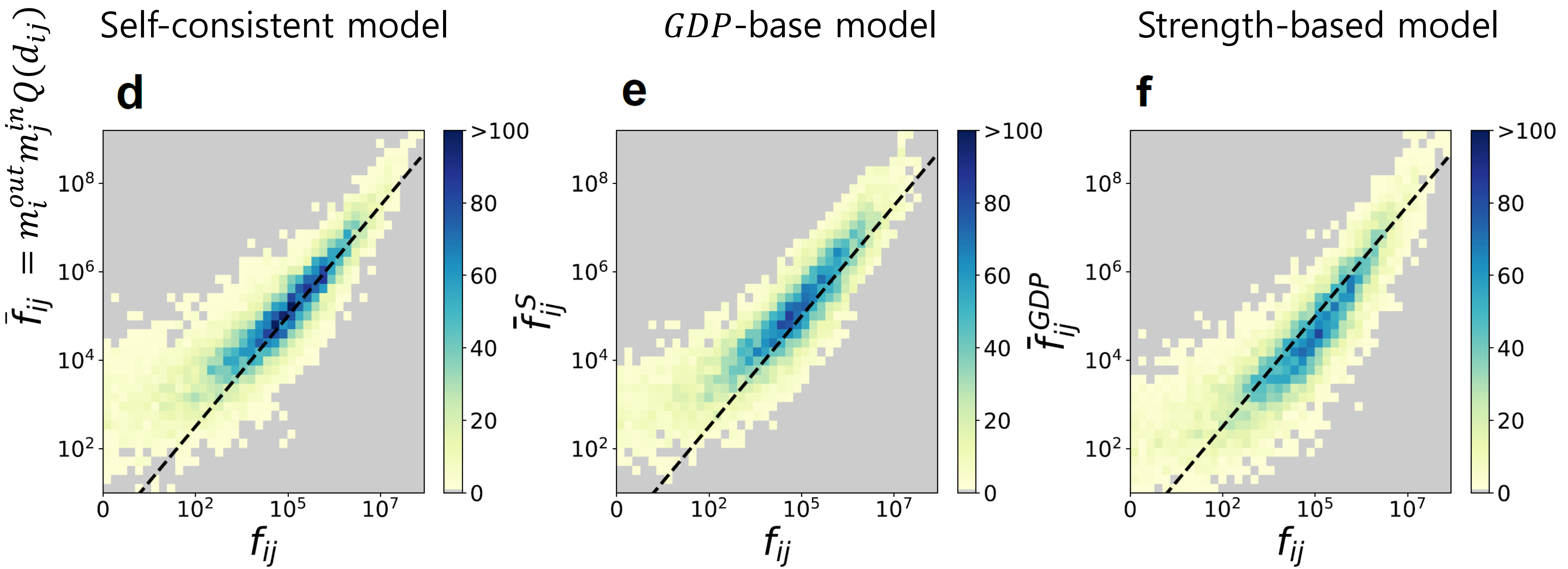
$$r_i^{in} = S_i^{in} / m_i^{in}$$



- Advantage index distribution for import
- Higher advantage index for Asian country

Model verification with real data

▪ Flow reconstruction



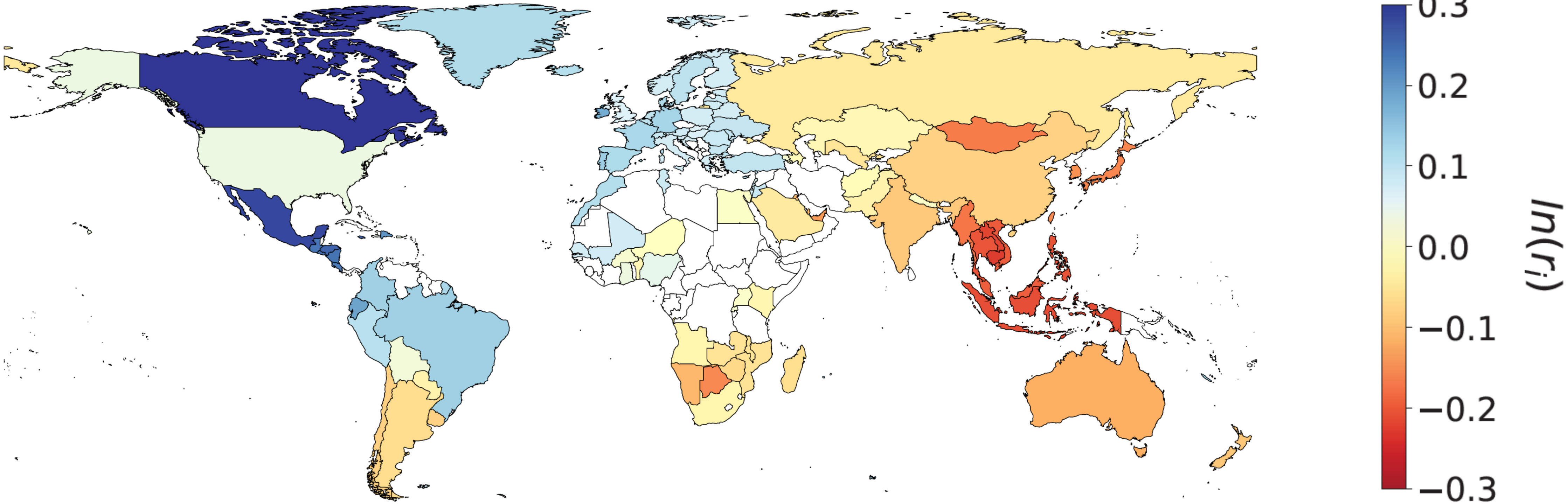
- Inference model successfully reconstruct the trade flow than strength model
- Overestimation for small flows

Model verification with real data

Result

- **Flow reconstruction**

$$r_i = \frac{r_i^{out}}{r_i^{in}} = \frac{S_i^{out}/m_i^{out}}{S_i^{in}/m_i^{in}} = \frac{\sum_j m_i^{in} Q(d_{ij})}{\sum_j m_j^{out} Q(d_{ij})}$$

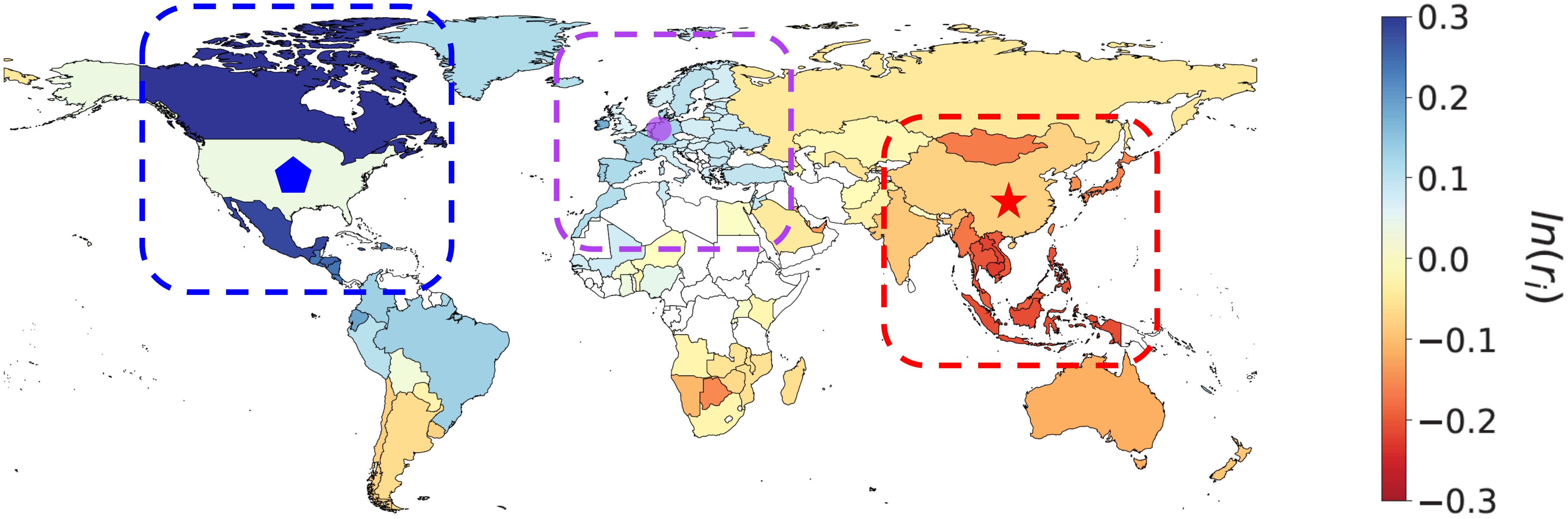


- Balance index: ratio of export/import advantage index
- Blue/Red: advantageous to export/import

Model verification with real data

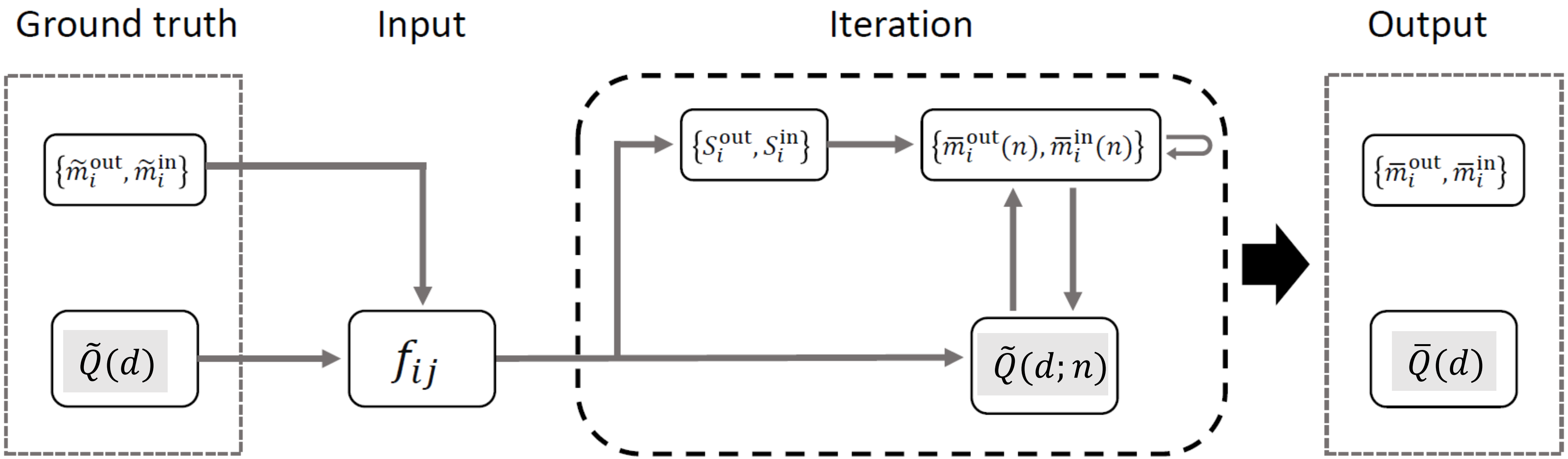
- **Flow reconstruction**

$$r_i = \frac{\sum_j m_i^{in} Q(d_{ij})}{\sum_j m_j^{out} Q(d_{ij})} \approx \frac{m_k^{in}}{m_k^{out}}$$



- Distinct pattern near three superpowers: USA, China, Germany
- Dependent on their economic characteristic: USA = import-centric, China = export-centric, Germany= balanced

Summary: self-consistent inference algorithm for gravity model



- Novel algorithm inferring the intrinsic characteristics of trade data
- Perfect to find the ground truth of synthetic network
- Distinguishing between the intrinsic capacity and economic environment of each country.

https://arxiv.org/abs/2106.10025