Extended supply-demand model in a distribution network

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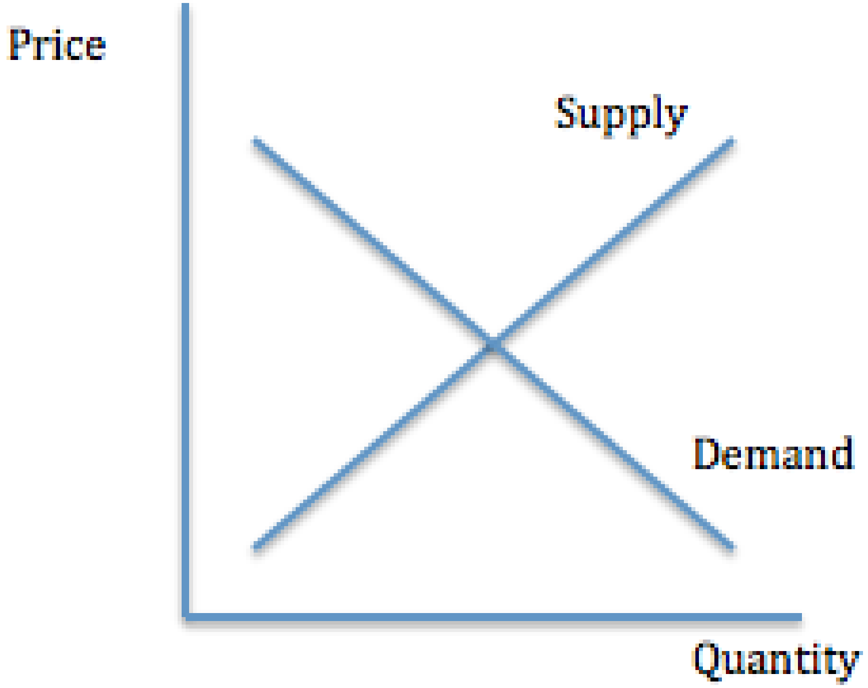
Introduction

Q: How does price change

when the supply of products changes?

Introduction

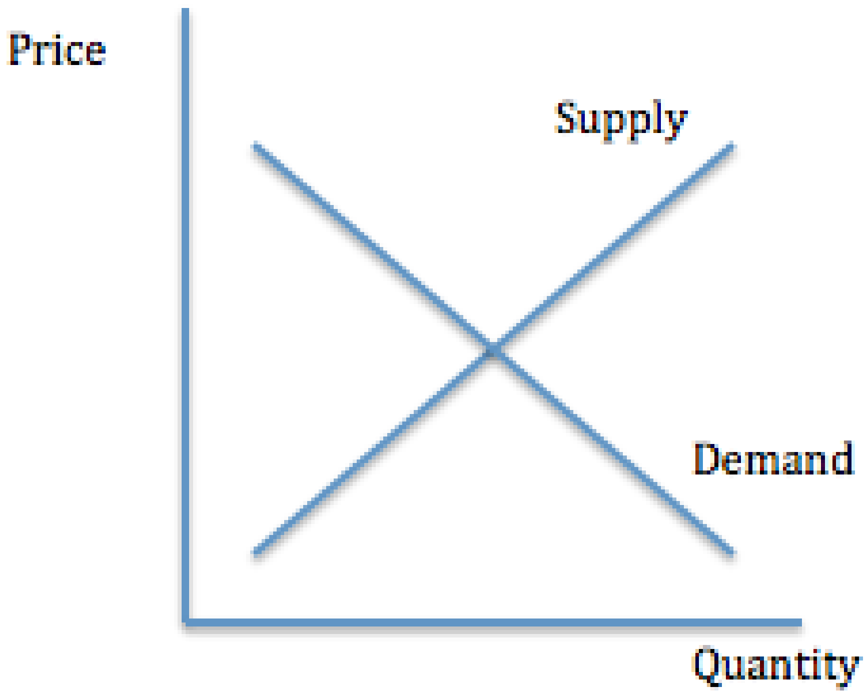
A1:Supply-demand theory



- Classical approach in microeconomics
- Price is determined by supply and demand

Introduction

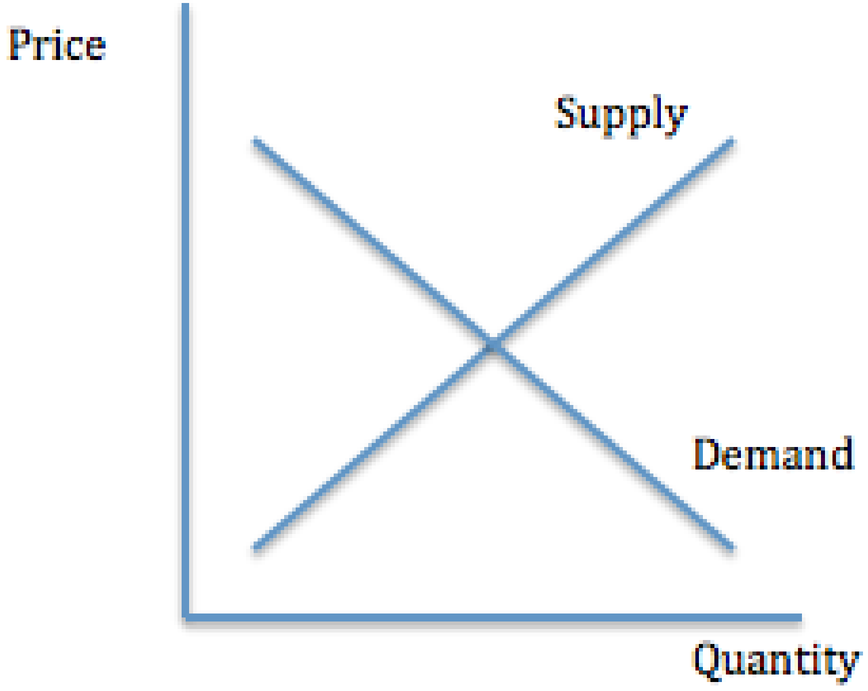
A1:Supply-demand theory



- Classical approach in microeconomics
- Price is determined by supply and demand
- **One-to-one relationship**

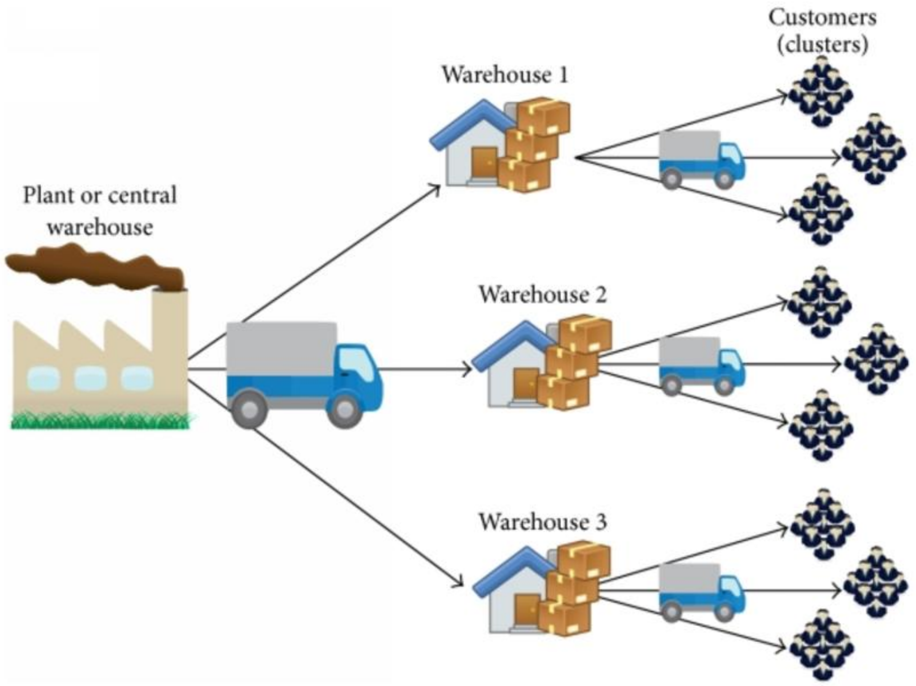
Introduction

A1: For one-to-one relationship



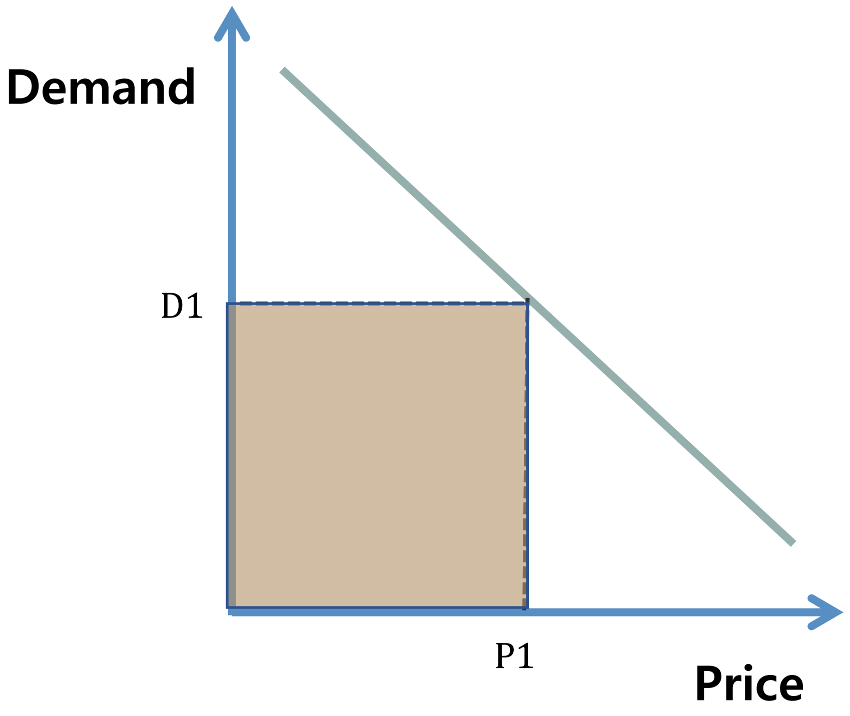
?

A2: For distribution network



Theory

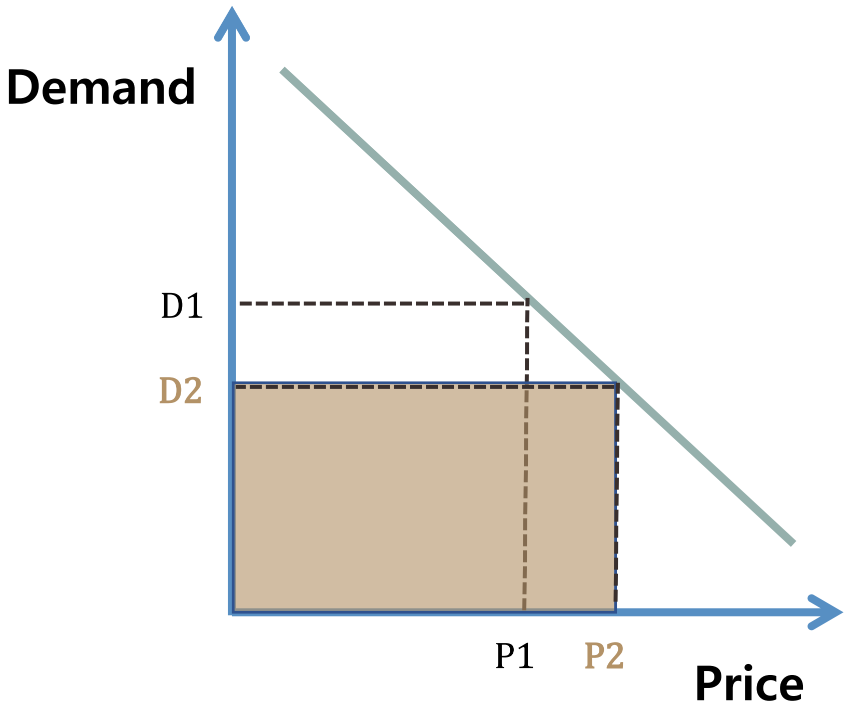
Price elasticity(ε)



Price	100$
Demand	100
Profit (Price×Demand)	10000$

Theory

Price elasticity(ε)

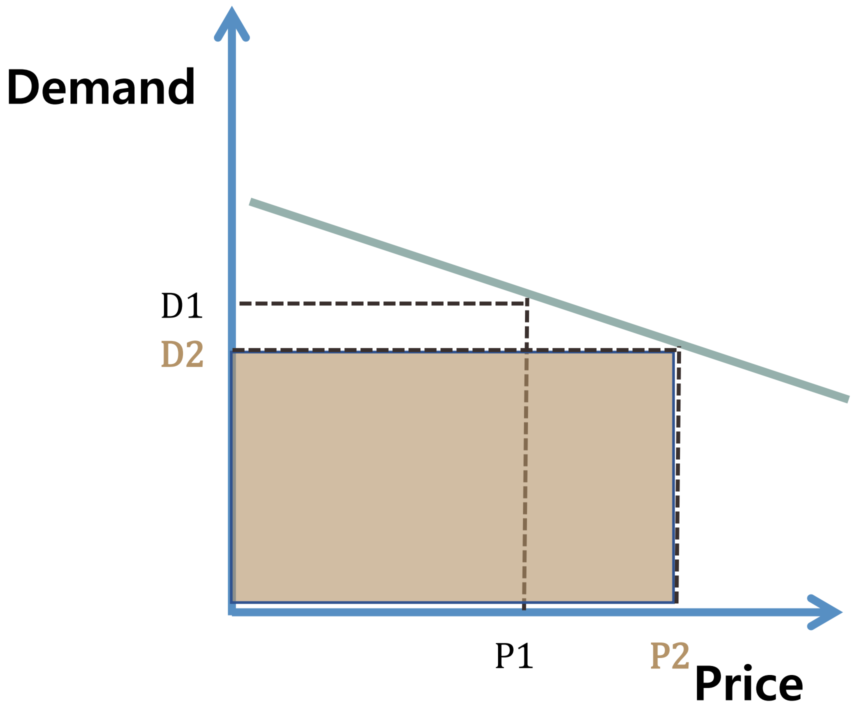


Price	100$	→	110$	=	100(1+0.1)$
Demand	100	→	(100 – 10ε)	=	100(1-0.1ε)
Profit (Price×Demand)	10000$	→	(11000 - 1100ε)$	=	10000(1+0.1)(1-0.1ε)$

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Theory

Price elasticity(ε)



When ε=0.5 (low)

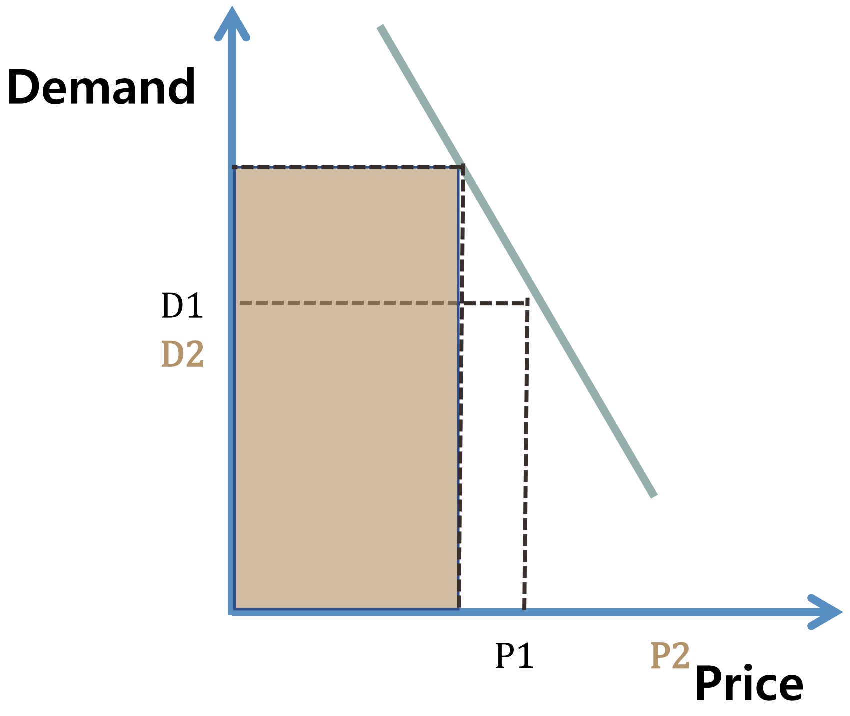
Price	100$	→	110$	=	100(1+0.1)$
Demand	100	→	95	=	100(1-0.05)
Profit (Price×Demand)	10000$	→	10450$	=	10000(1+0.1)(1-0.05)$

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Seller raise the price

Theory

Price elasticity(ε)



When ε=2 (high)

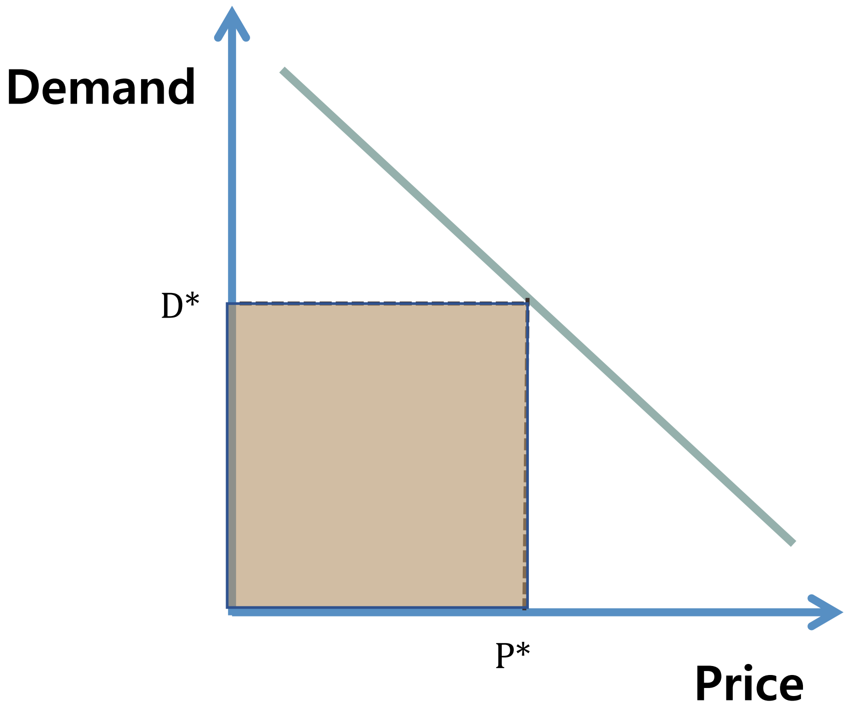
Price	100$	→	95$	=	100(1-0.05)$
Demand	100	→	110	=	100(1+0.1)
Profit (Price×Demand)	10000$	→	10450$	=	10000(1-0.05)(1+0.1)$

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Seller lower the price

Theory

Price elasticity(ε)



When ε=1

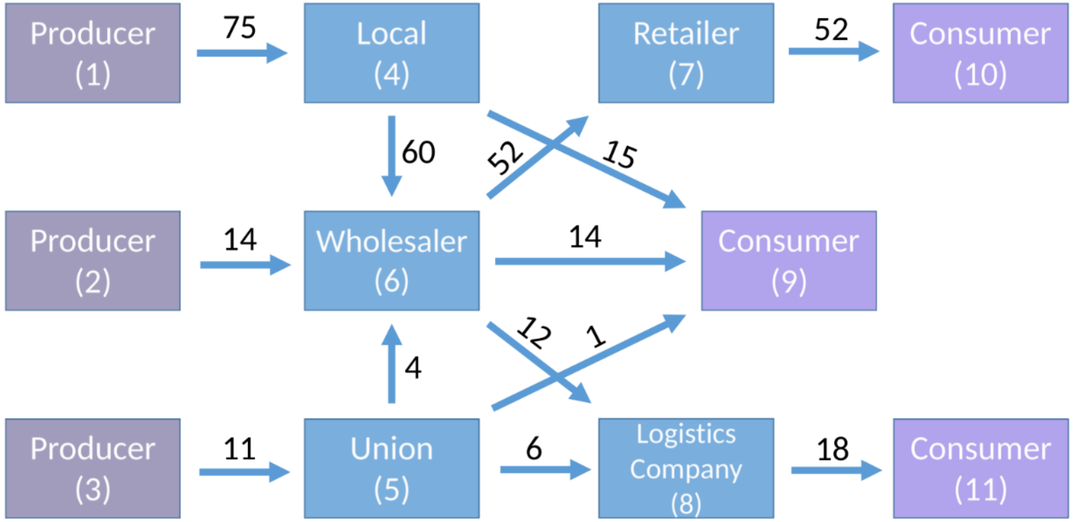
Price	100$	→	110$	=	100(1+0.1)$
Demand	100	→	90	=	100(1-0. 1)
Profit (Price×Demand)	10000$	→	9900$	=	10000(1+0.1)(1-0.1)$

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Balanced price
(Nash equilibrium)

Theory

But, in a distribution network..

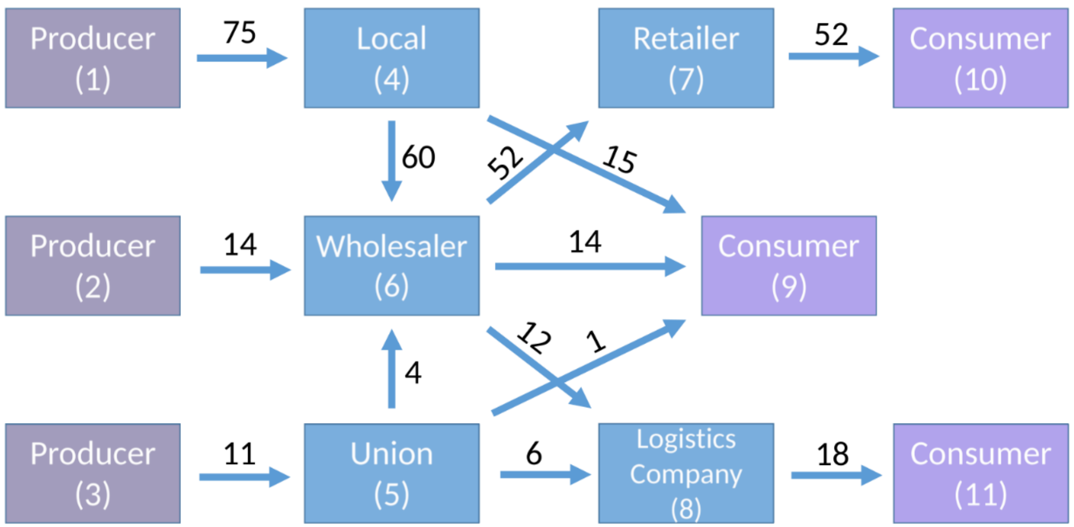


Korean cabbage distribution network(2014)

- Multiple producer and consumer
- Products flow in distribution network
- Complex network structure
- **Difficult to find equilibrium**

Theory

But, in a distribution network..



Korean cabbage distribution network(2014)

Node

(Price) Producer : Making product flow

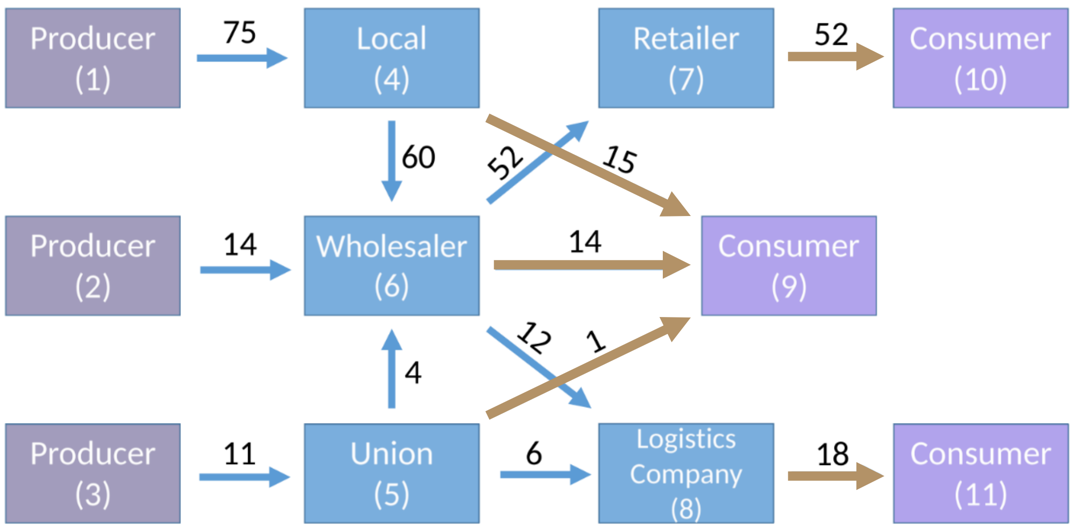
(Price) Distributor : Mediating product flow

(Demand) Consumer : Consuming product flow

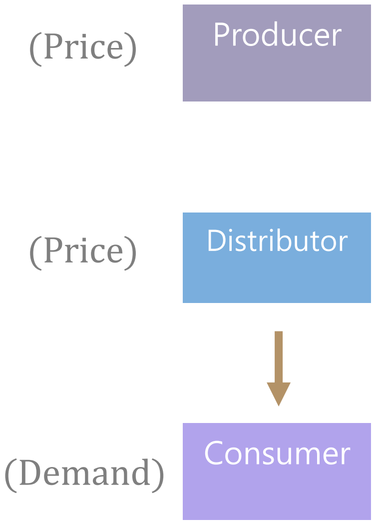
Theory

But, in a distribution network..

Node



Korean cabbage distribution network(2014)

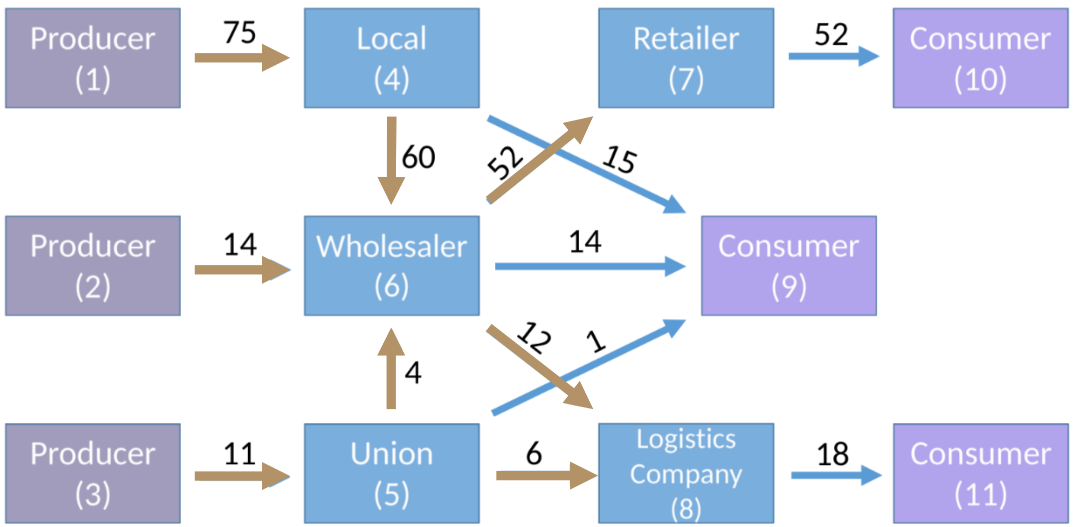


$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Only available on
Distributor-consumer links

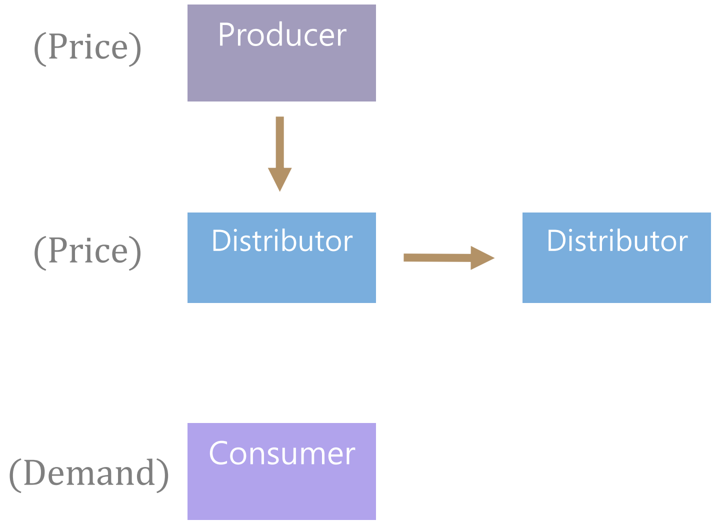
Theory

But, in a distribution network..



Korean cabbage distribution network(2014)

Node



What is the function of distributor?

Theory

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Demand **Price**

Node

$\boldsymbol{i}$ $\boldsymbol{j}$

(Price) Distributor → Consumer

or

(Price) Distributor → Distributor

Producer → Distributor

→Consumer's demand is determined by **Price**

Theory

$$\frac{\Delta D}{D} = -\varepsilon \frac{\Delta P}{P}$$

Demand

Price

Price difference

$$\boxed{\frac{\Delta D}{D} = \varepsilon \frac{\Delta (P_j - P_i)}{(P_j - P_i)}}$$

Node

	$\boldsymbol{i}$		$\boldsymbol{j}$
(Price)	Distributor	→	Consumer

or

(Price)	Distributor	→	Distributor
	Producer	→	Distributor

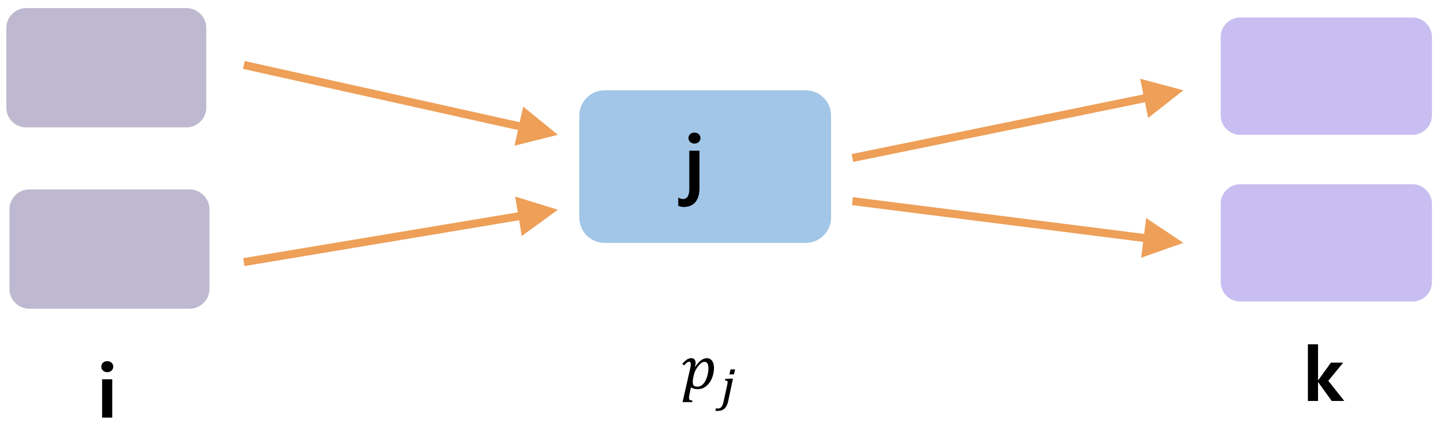
→Distributor's demand is determined by

Price difference (profit per product)

Theory

Continuity

Each distributor node must minimize its stock,



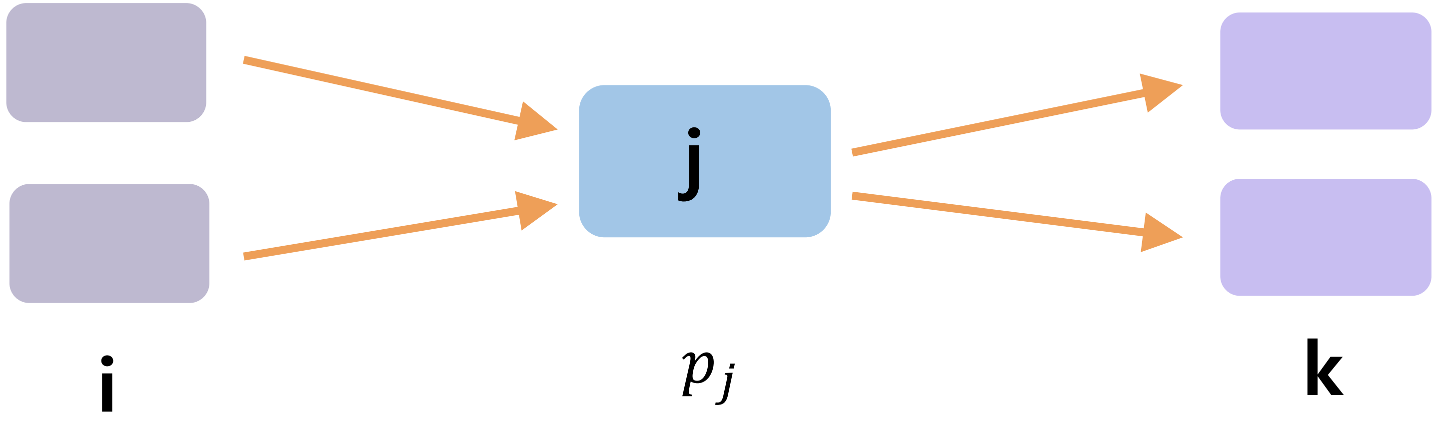
(Flow difference) $\Delta f_j = \sum_i \Delta D_{ij} - \sum_k \Delta D_{jk} = 0$

Theory

Continuity

Each distributor node must minimize its stock,

By adjusting the price p_j



(Flow difference) $\Delta f_j = \sum_i \Delta D_{ij} - \sum_k \Delta D_{jk} = 0$

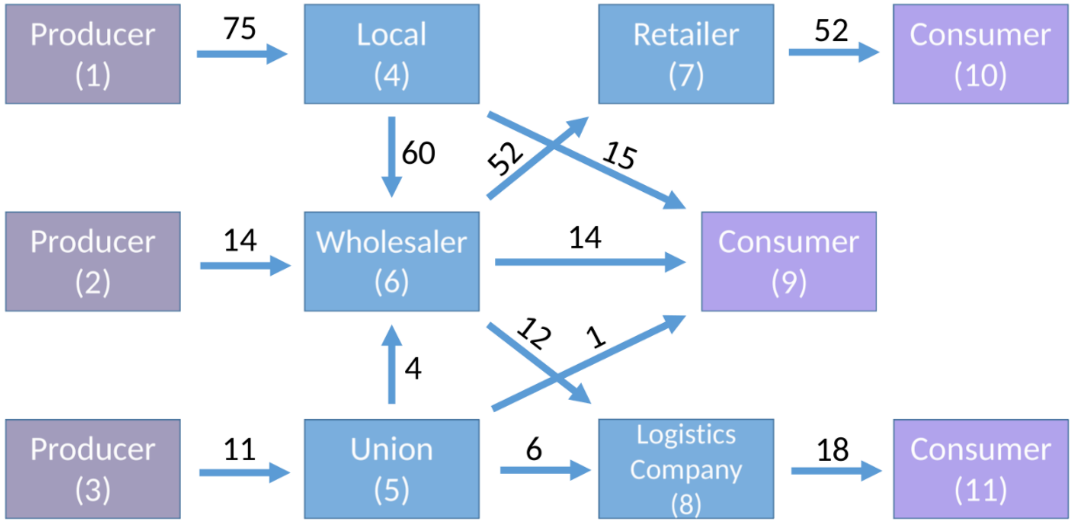
$$\rightarrow \sum_i \frac{\lambda_{ij}\Delta(p_j - p_i)}{p_j - p_i} - \sum_k \frac{\lambda_{jk}\Delta(p_k - p_j)}{p_k - p_j} = 0$$

$$\boxed{\rightarrow \Delta p_j = \frac{\Delta f_j}{\sum_i \frac{D_{ij}\lambda_{ij}}{p_j - p_i} + \sum_k \frac{D_{jk}\lambda_{jk}}{p_k - p_j}}}$$

(Adjusting Price)

Simulation

Setting



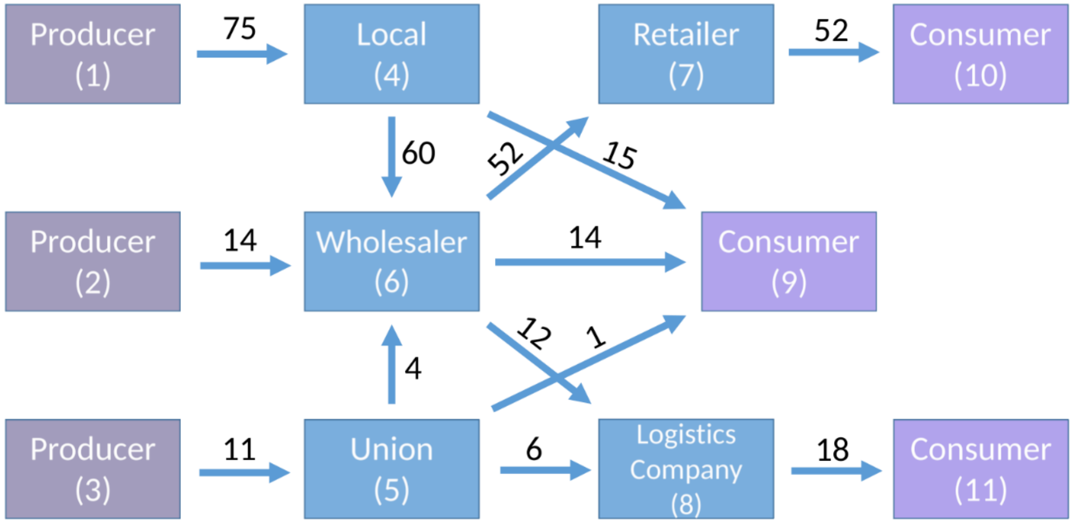
Korean cabbage distribution network(2014)

- Assume that initial network is stable
- Each node has own price (except consumer)
- Prices change upon supply change

node	price
1,2,3	1
4,5	2
6	3
7,8	4

Simulation

Algorithm



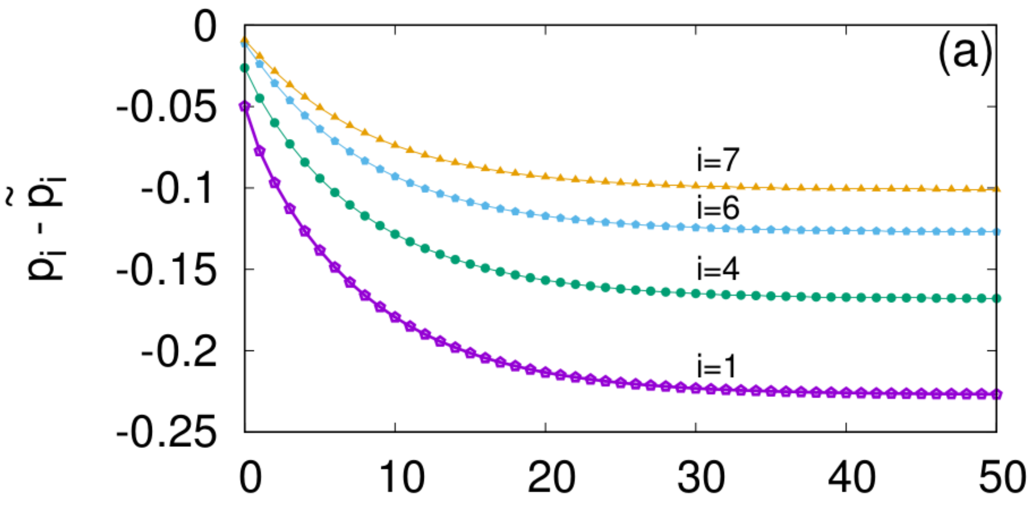
Korean cabbage distribution network(2014)

1. Apply a small supply change
2. Determine the price on each node
3. Iterate until all nodes become stable

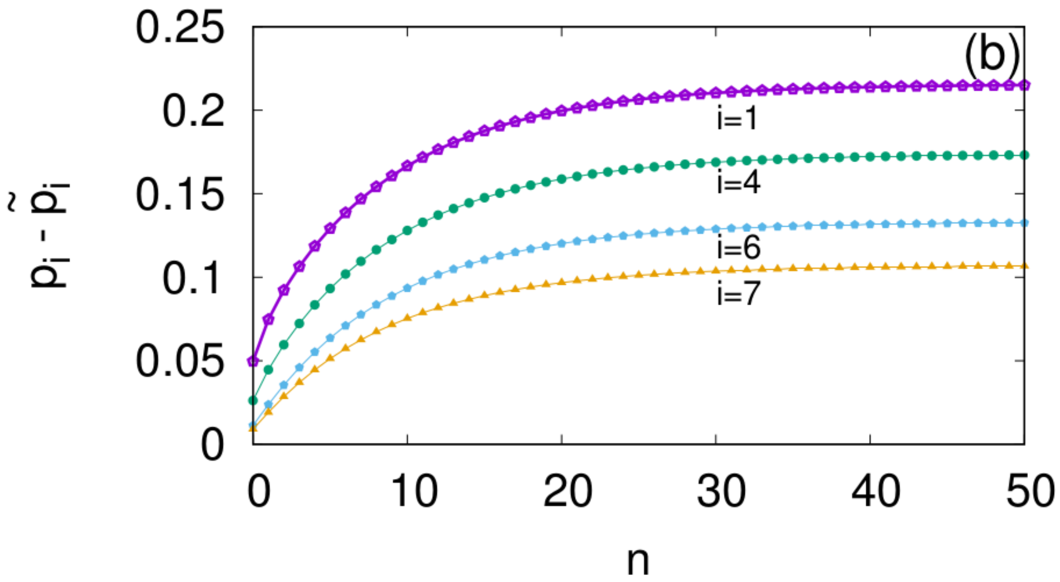
Result

1.Price change in time

$\alpha_1 : 5\% \ increased$



$\alpha_1 : 5\% \ decreased$

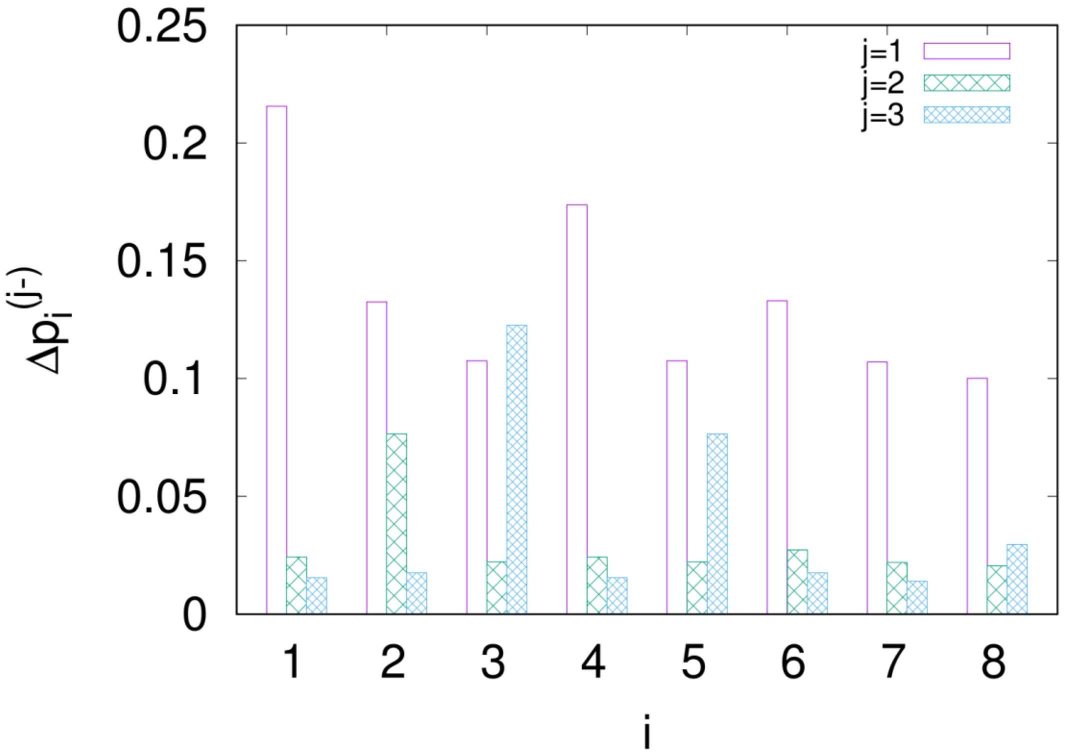


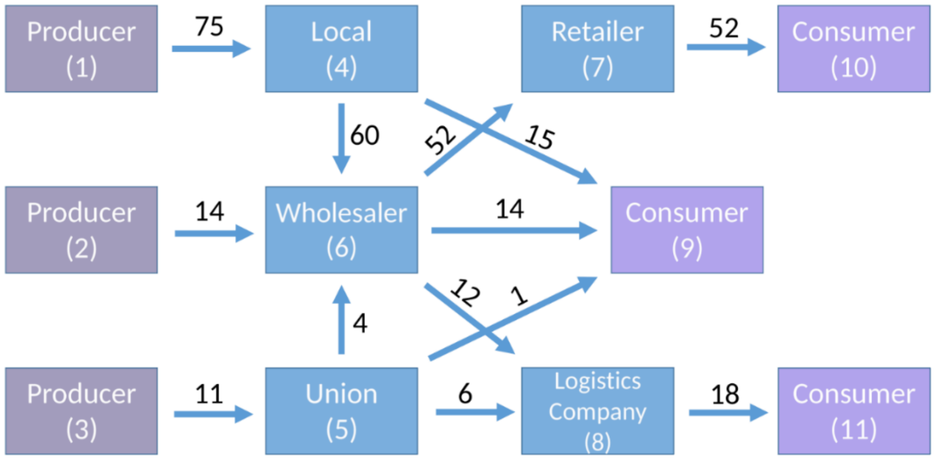
New equilibrium prices emerge!

Result

2.Price change with various fluctuation sources

α_j: 5% decreased

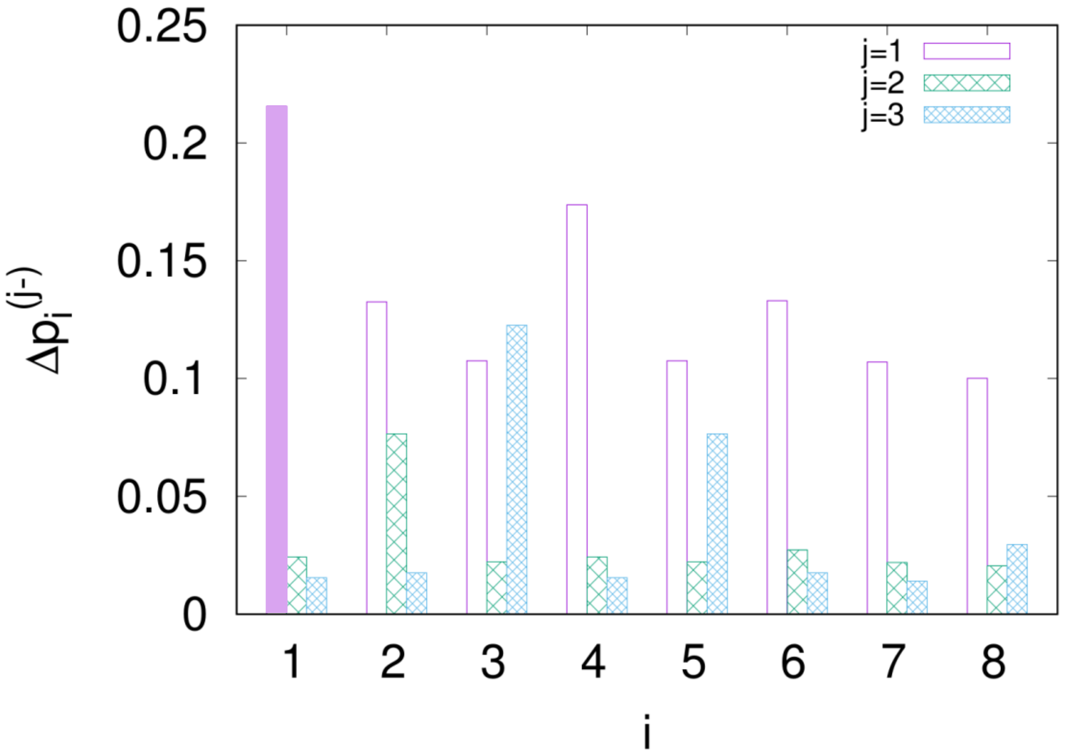


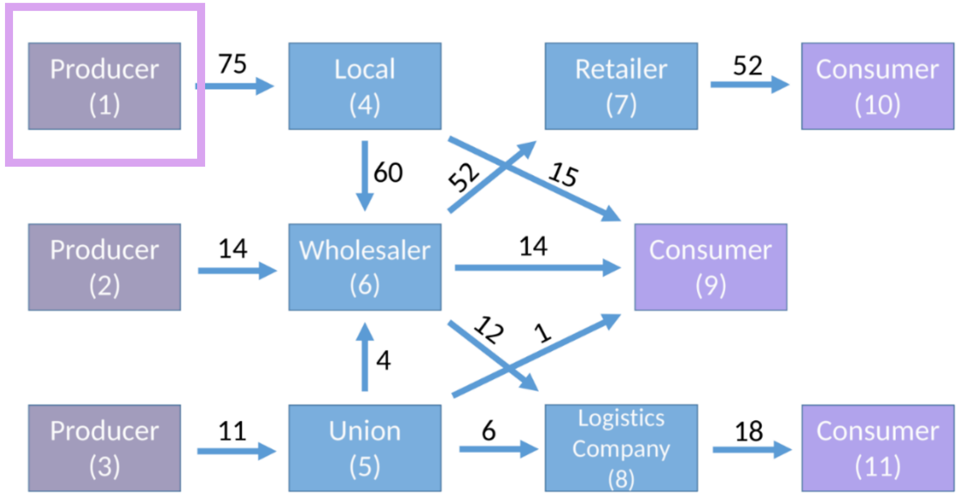


Result

2.Price change with various fluctuation sources

$\alpha_j: 5\%\ decreased$





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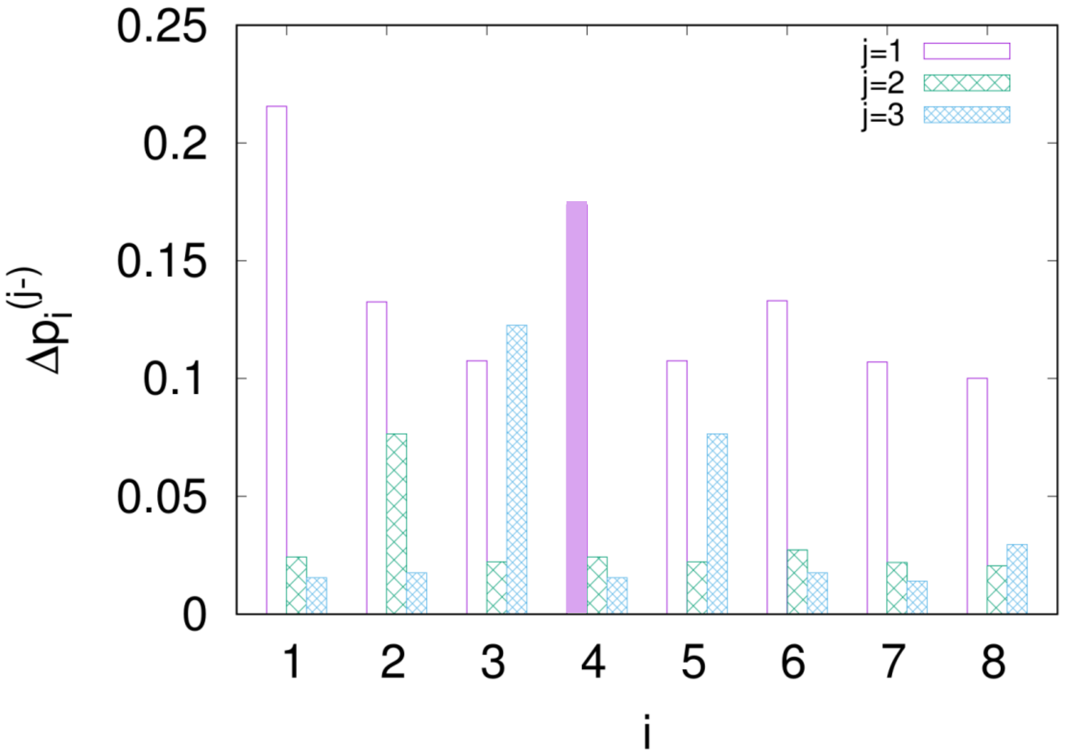

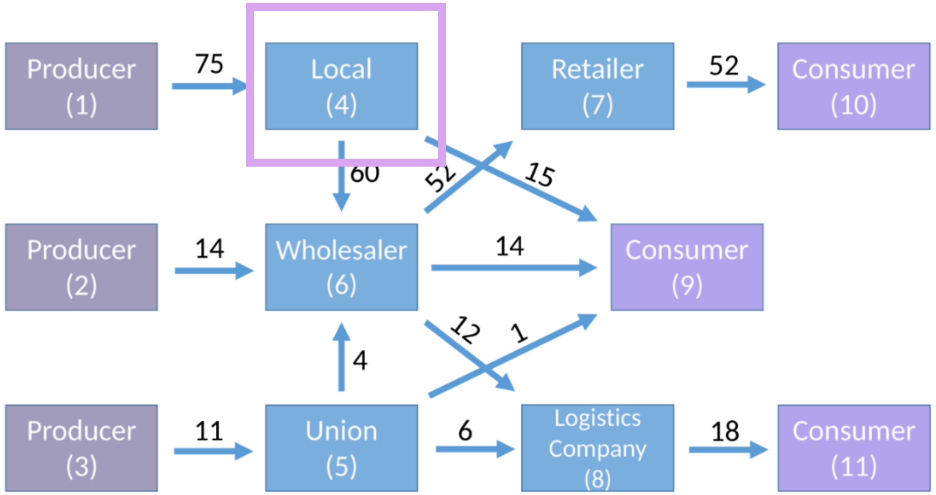

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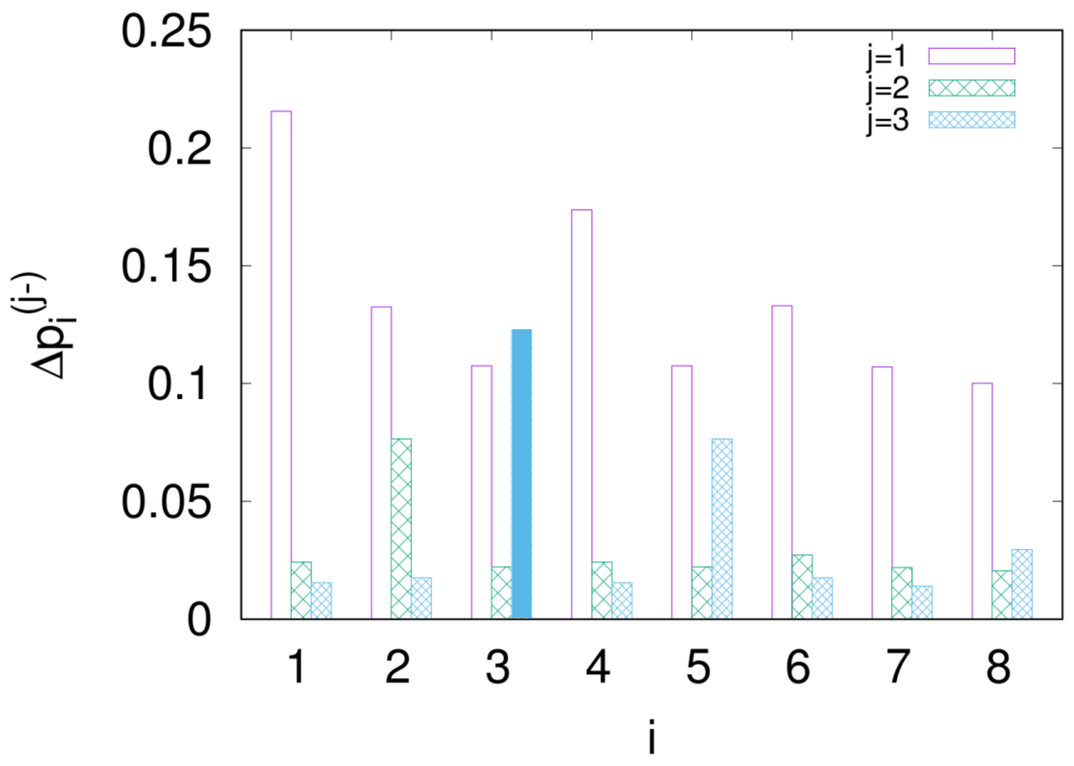

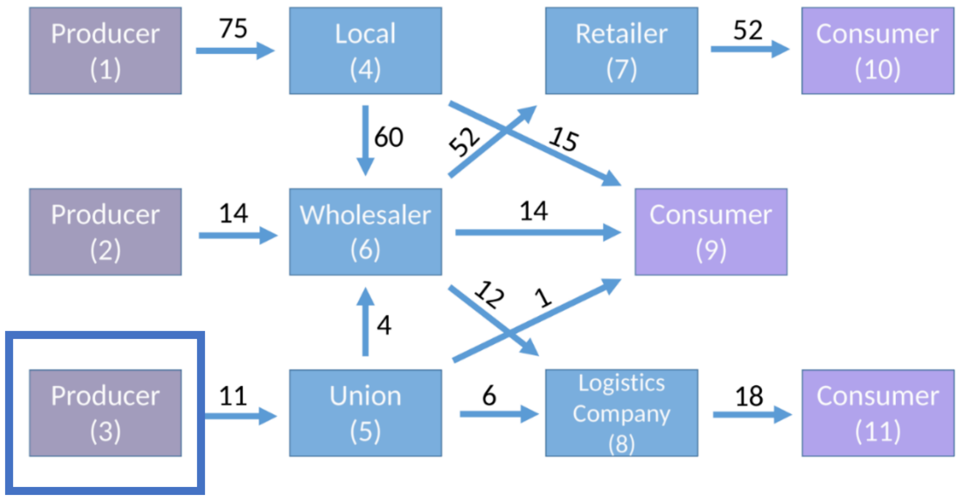

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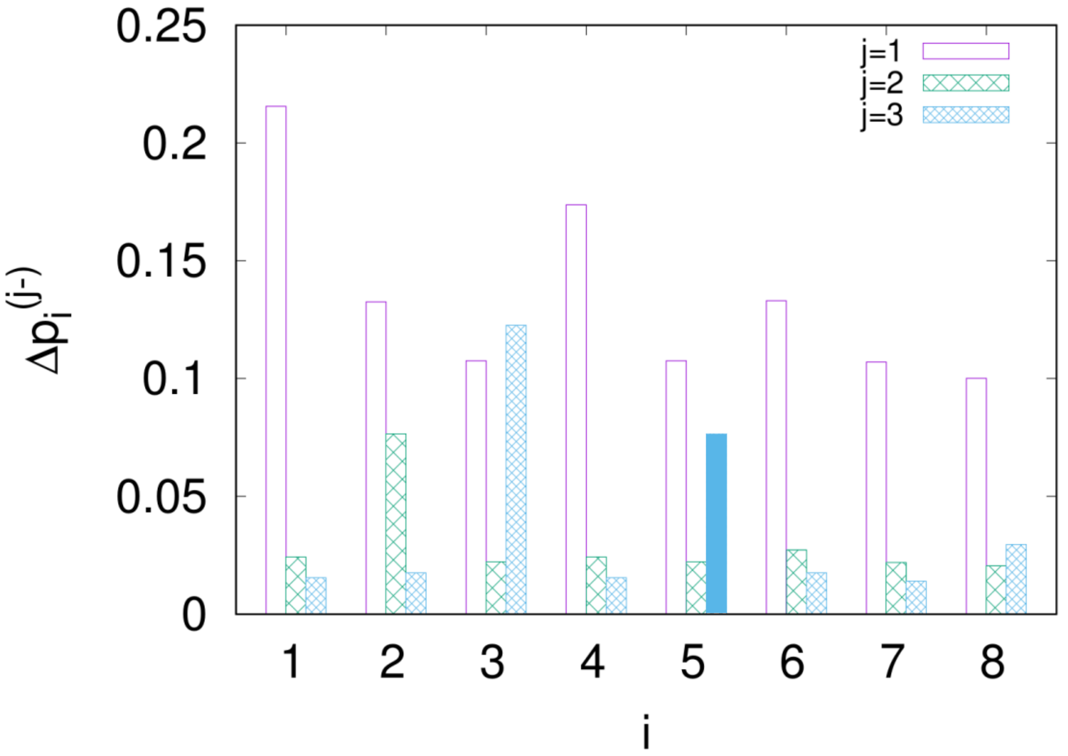

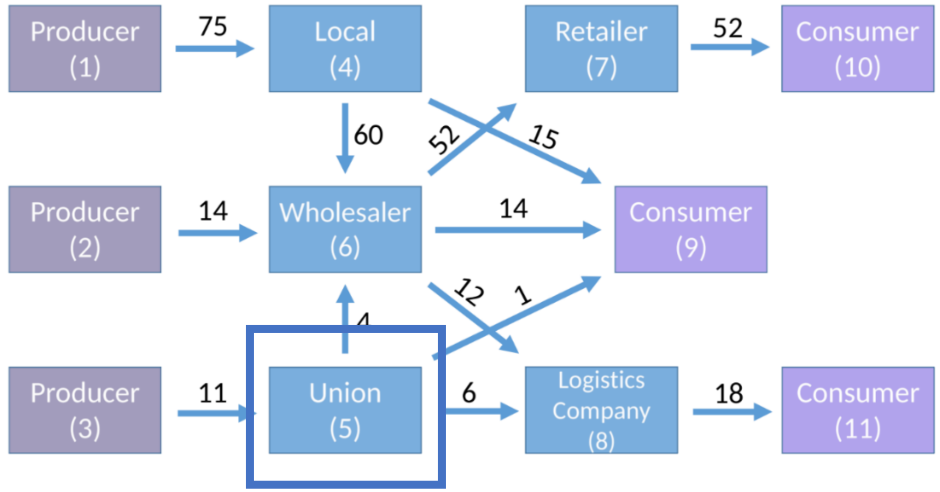

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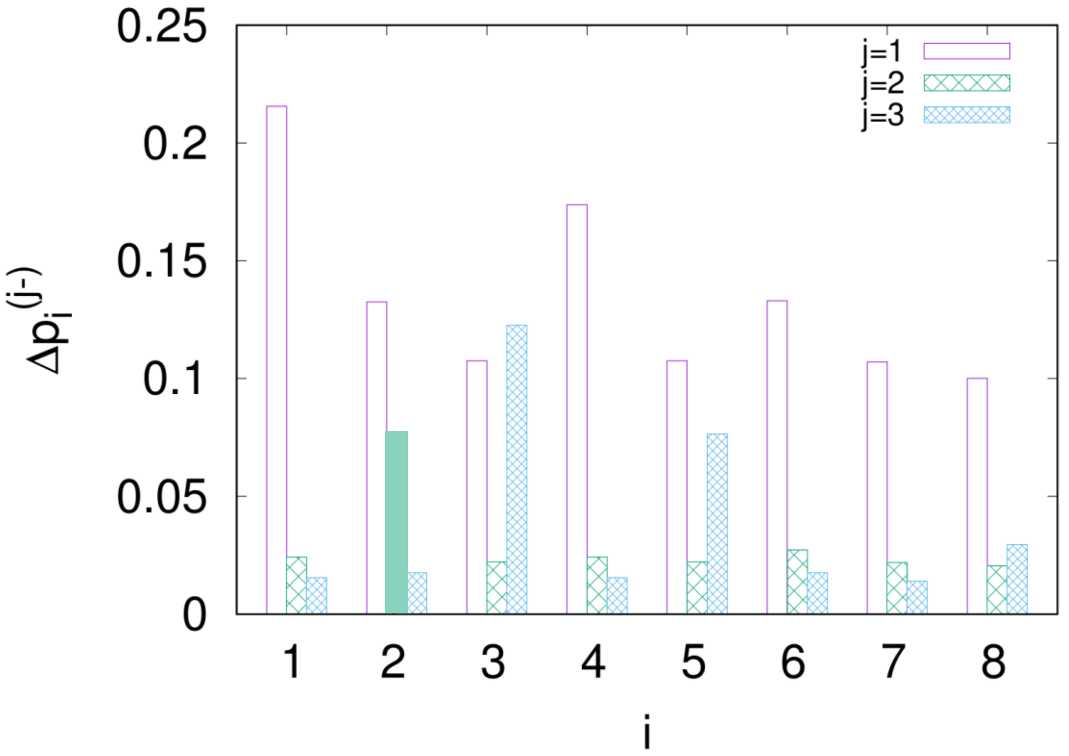

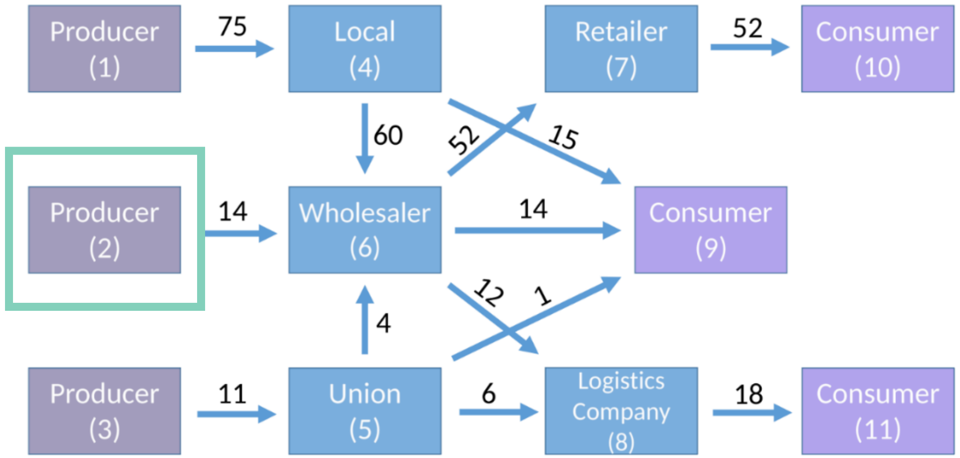

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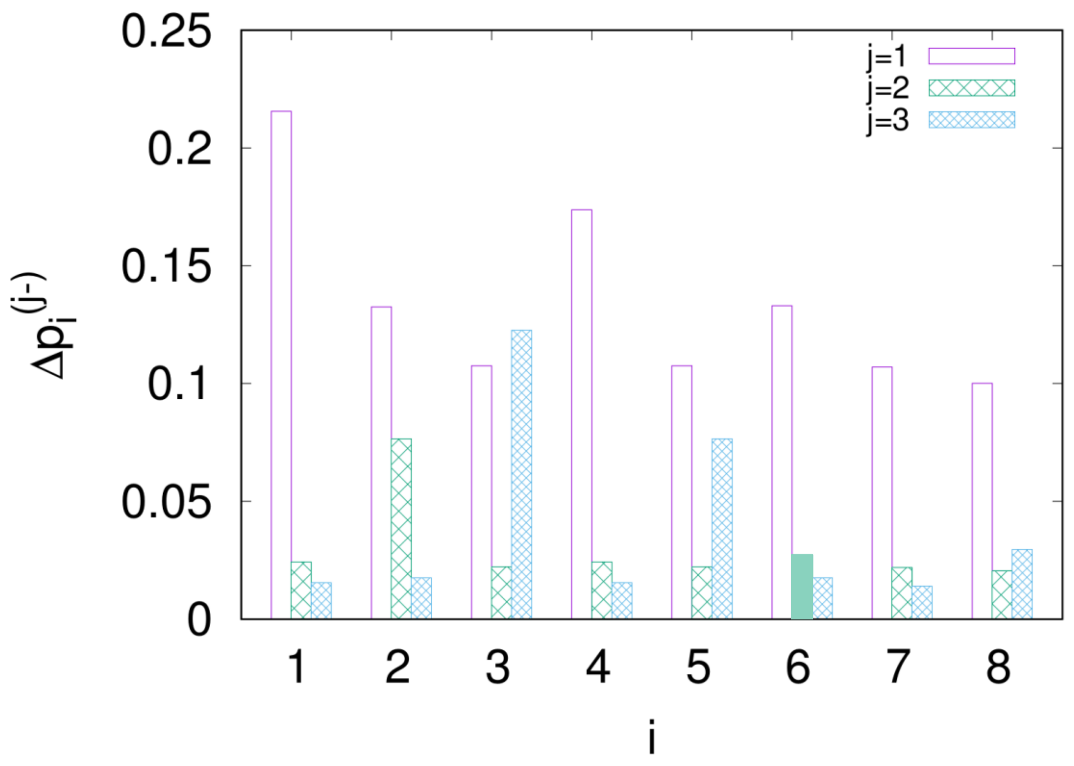

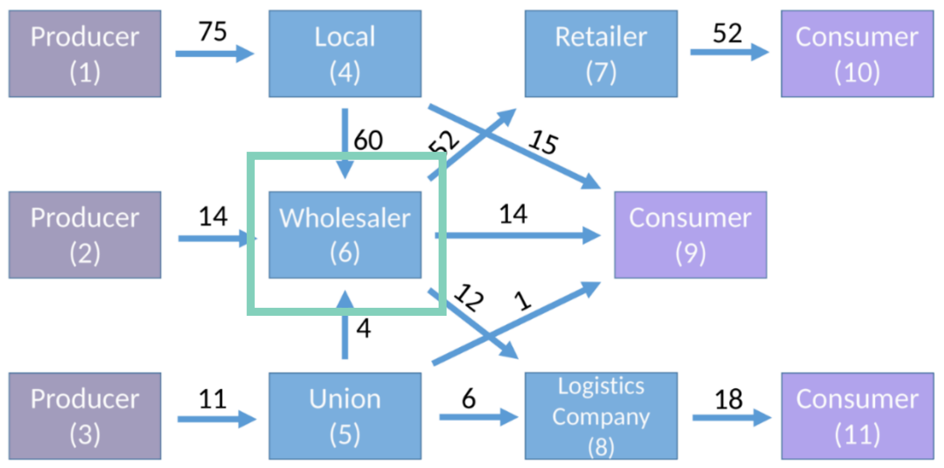

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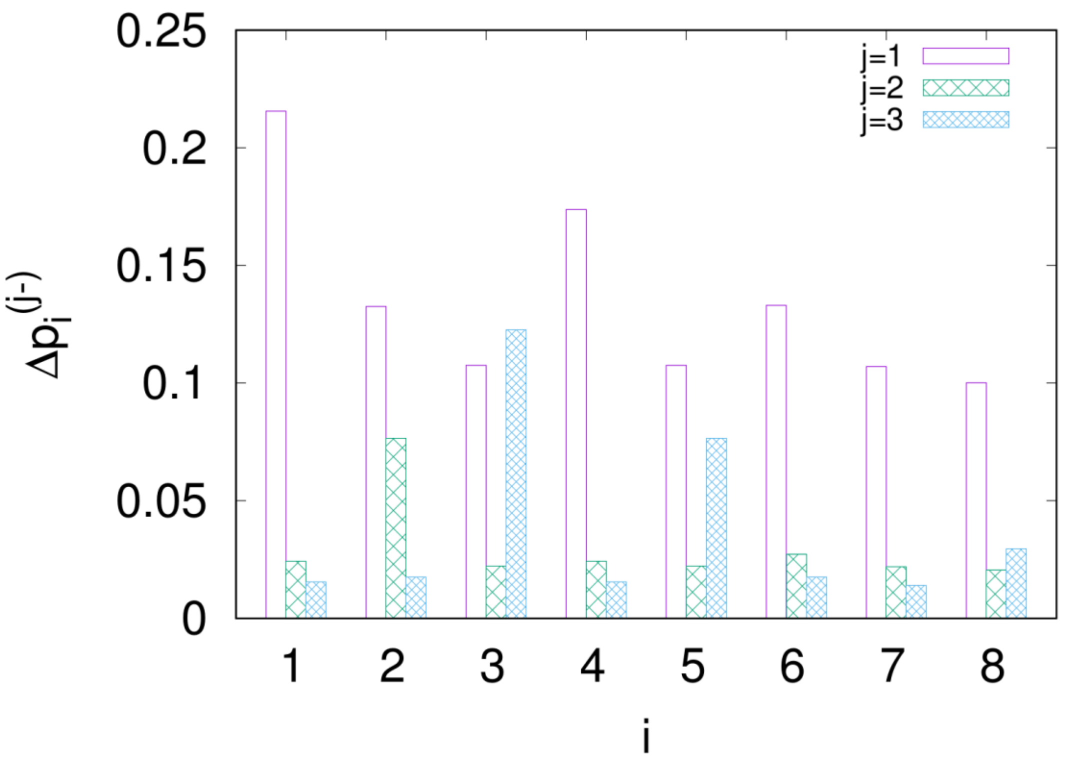

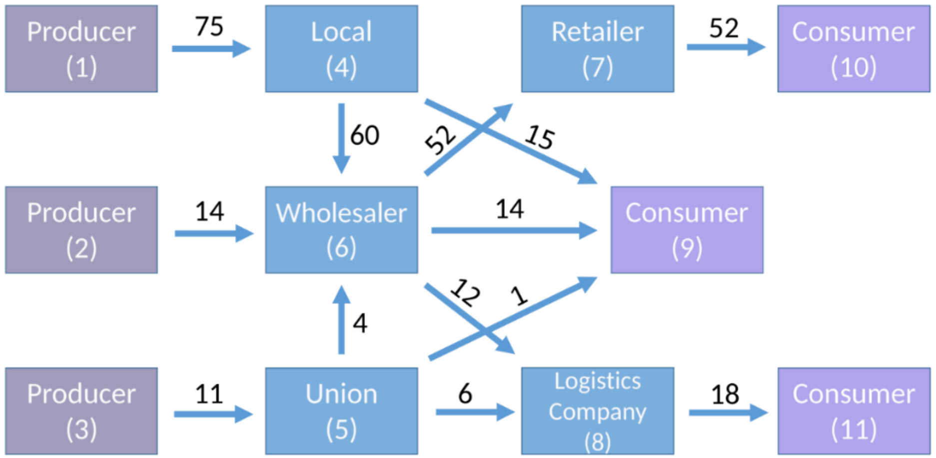

Impact of supply change depends on network structure

Summary & Limitation

Summary

- Extended supply-demand theory
- Applied to complex distribution network
- Simulation of price change in real network
- New equilibrium condition emerges
- Importance of network structure

Limitation

- Distributors do not satisfy Nash equilibrium
- Abrupt price change not observed

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