COMPLEX POWER SYSTEM RESEARCH GROUP

# The multi-scale landscape of community inconsistency in networks

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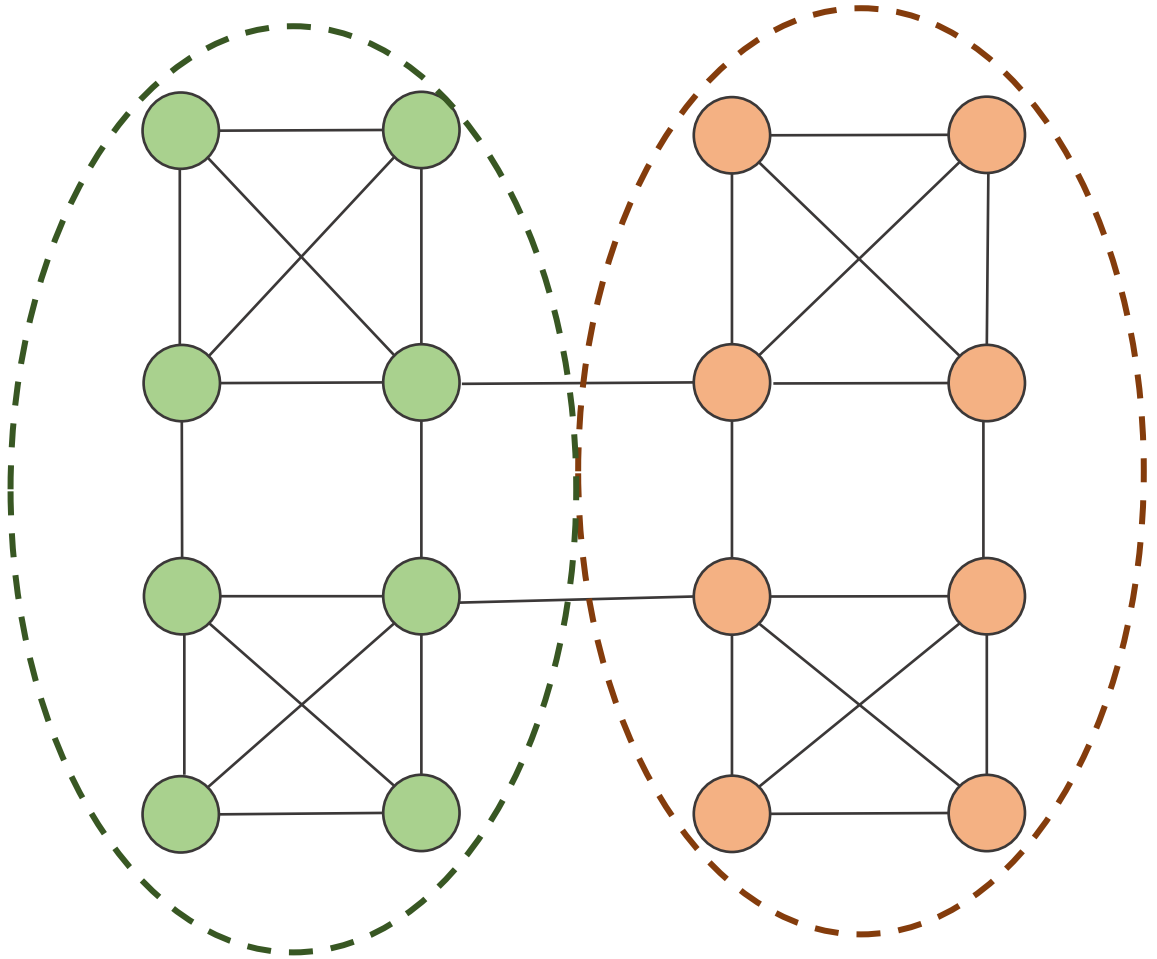
# Community: Mesoscopic modular structure





Modularity

Null model

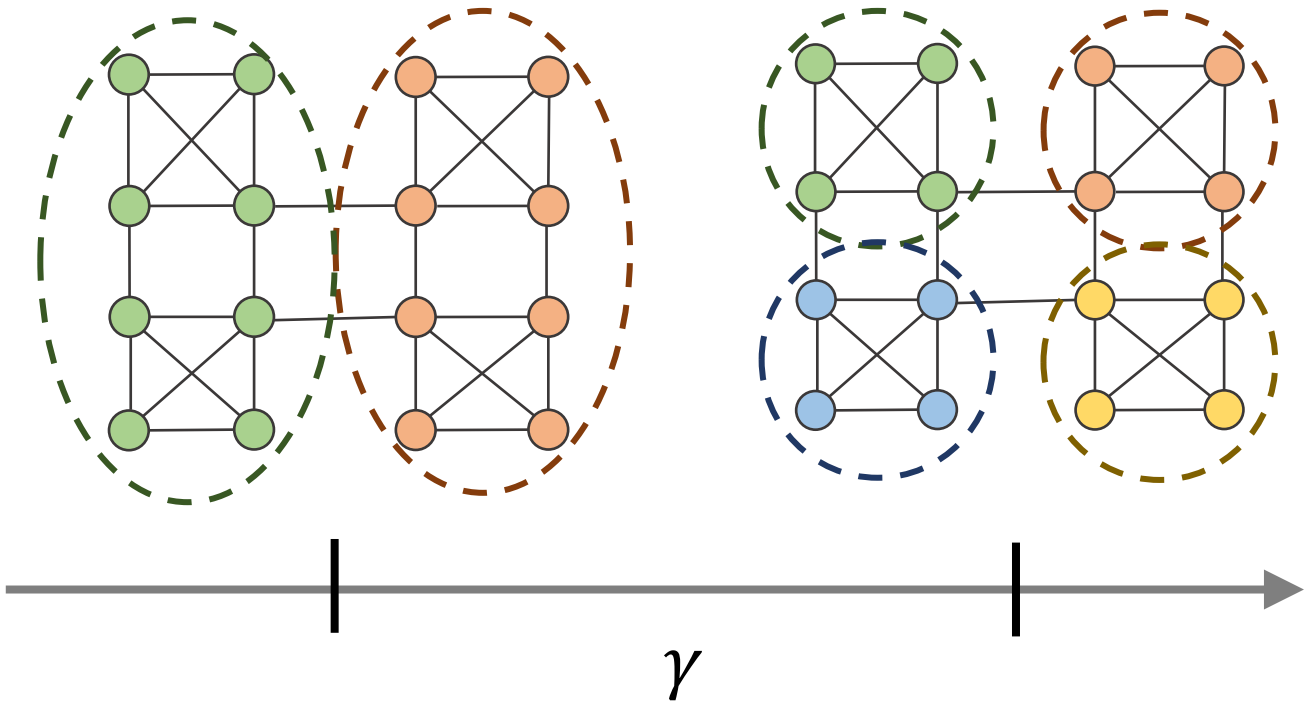
$$\mathcal{H} = \frac{1}{2m} \sum_c \left( e_c - \gamma \frac{K_c^2}{2m} \right)$$

Number of intra-community links

- Community: Densely connected group of nodes
- Found by the maximization of object function
- **Varying results in multiple realizations**

# Resolution problem of community detection






Modularity

$$\mathcal{H} = \frac{1}{2m}\sum_c \left( e_c - \gamma \frac{K_c^2}{2m} \right)$$

Resolution parameter

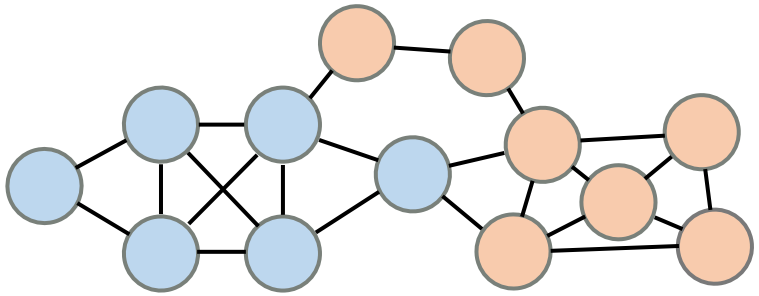
- γ: resolution parameter of community structure
- **No criterion to determine the proper resolution**

# Community ensemble



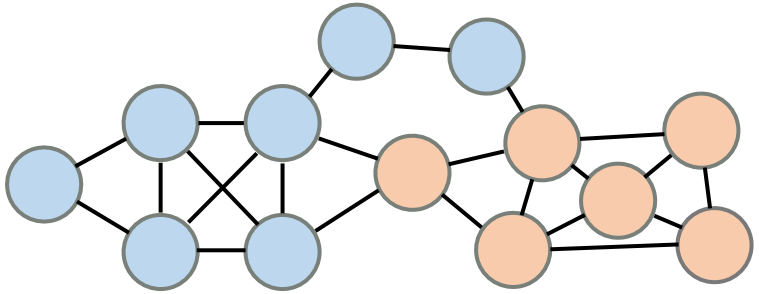
*3) Sci. Rep. 9 (2019) 8574*

$P(C_1) = 0.6$



How similar? → $S_{12}$ $(0\sim1)$[3)]

$P(C_2) = 0.4$

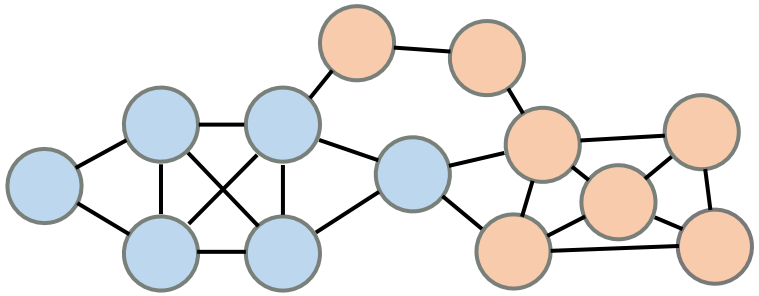


- Ensemble of community detection results from multiple realizations
- How diverse are the configurations in community ensemble?
- Using the element-centric similarity (EC) between each configuration pair

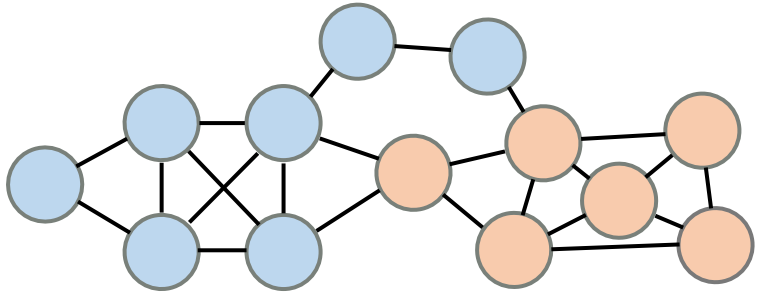
# Partition Inconsistency (Ω)



$P(C_1) = 0.6$

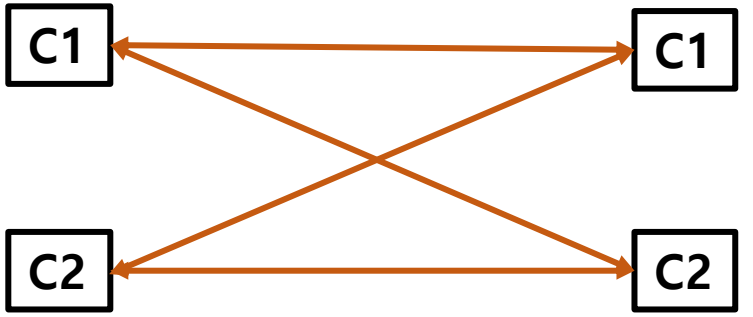


How similar? → $S_{12}$ (0~1)



$P(C_2) = 0.4$

$S_{\alpha\beta}$




$$\Omega = \frac{1}{\langle S_{\alpha\beta} \rangle} = \left[ \sum_{\alpha} \sum_{\beta} P(C_\alpha) P(C_\beta) S_{\alpha\beta} \right]^{-1}$$

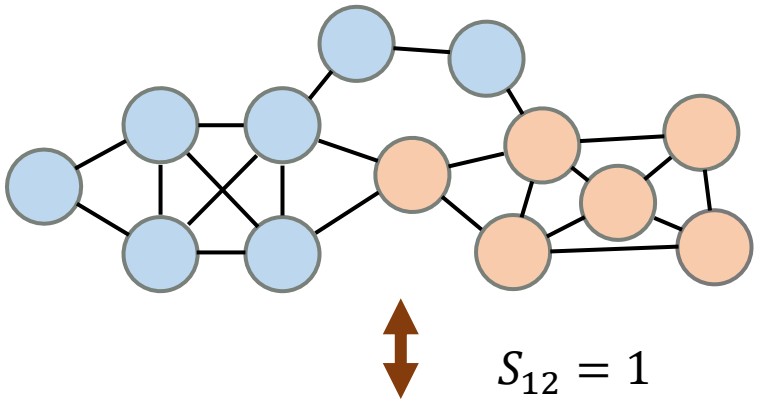
$$= \frac{1}{P(C_1)^2 + P(C_2)^2 + 2P(C_1)P(C_2)S_{12}}$$

- Partition inconsistency: A measure to assess the stability of community ensemble
- Inverse of average similarity between all the configuration pairs

# Partition Inconsistency (Ω)



$P(C_1) = 0.6$




$P(C_2) = 0.4$

$If\ S_{12} = 1$

$$\Omega = \frac{1}{\langle S_{\alpha\beta} \rangle} = \left[ \sum_{\alpha} \sum_{\beta} P(C_\alpha) P(C_\beta) S_{\alpha\beta} \right]^{-1}$$

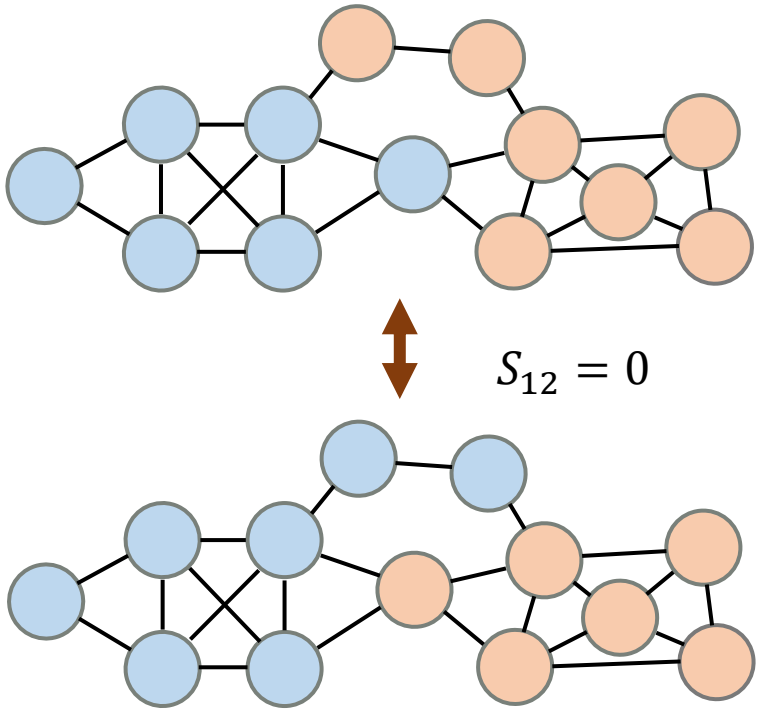
$$= \frac{1}{\left( P(C_1) + P(C_2) \right)^2} = 1$$

- If all the configurations are same → $\Omega = 1$ (minimum)

# Partition Inconsistency (Ω)



$$P(C_1) = \frac{1}{2}$$




$$P(C_2) = \frac{1}{2}$$

$$if\ S_{\alpha\beta} = 0\ for\ all\ distinct\ pairs$$

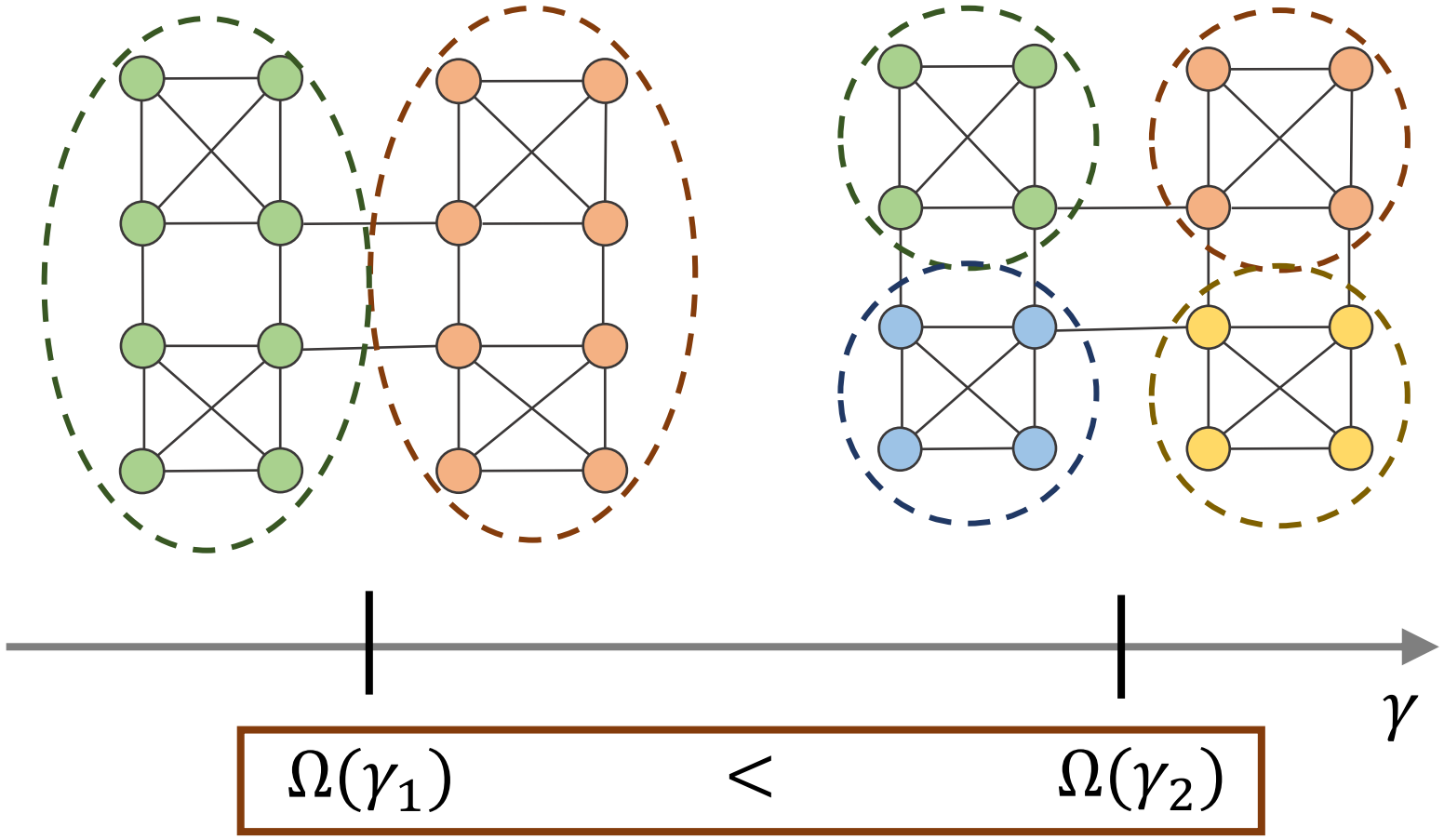
$$\Omega = \frac{1}{\langle S_{\alpha\beta} \rangle} = \left[ \sum_{\alpha} \sum_{\beta} P(C_\alpha) P(C_\beta) S_{\alpha\beta} \right]^{-1}$$

$$= \frac{1}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2 \times \left(\frac{1}{2}\right)^2} = \mathbf{2}$$

- If all the configurations are identical ($S = 0$) and appear with equal probability
- $\Omega$ = the number of configurations
- **Partition Inconsistency denotes the effective number of configurations in community ensemble**

# Consistency approach to resolution problem

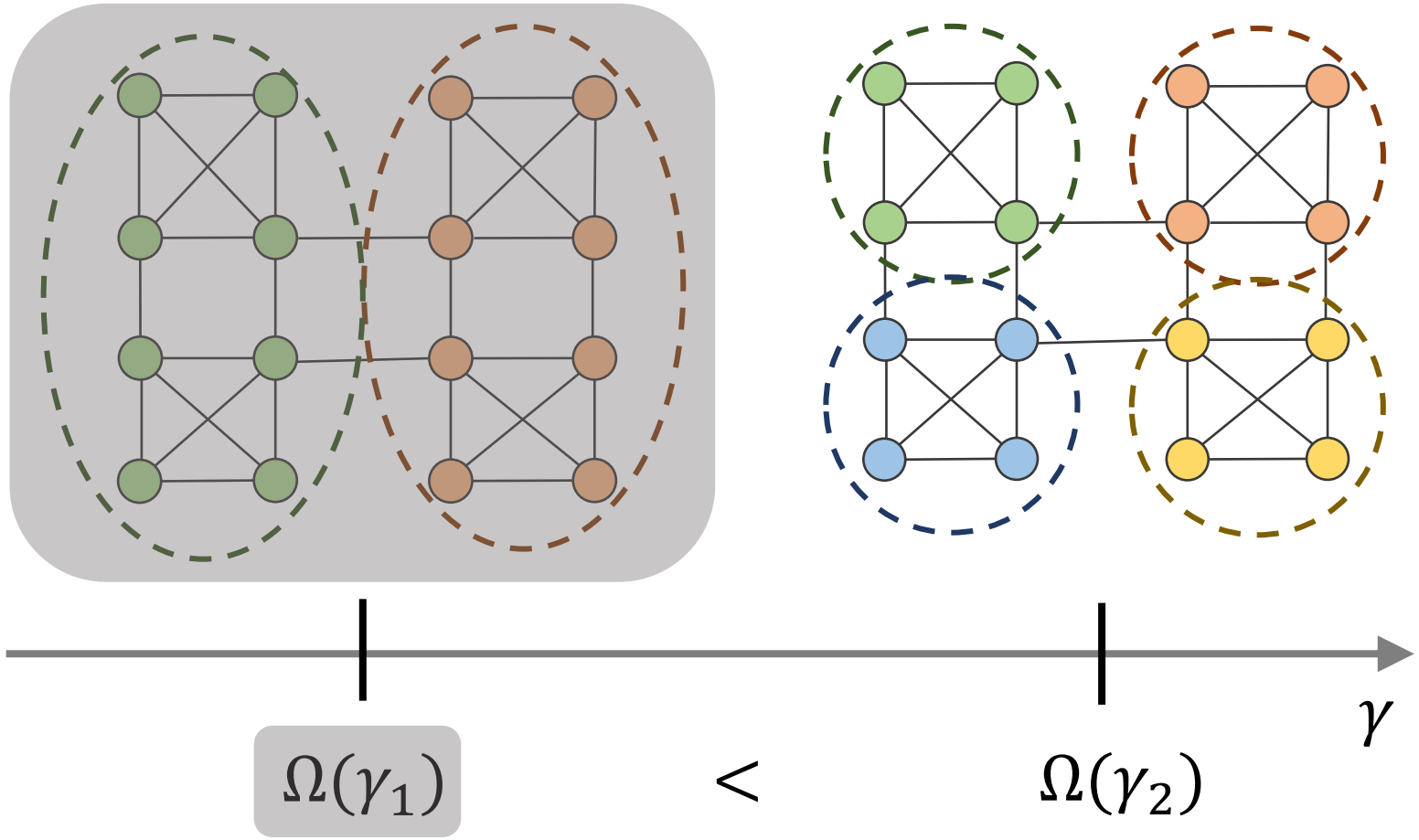





- Resolution of community structure can be determined by Partition Inconsistency
- Comparison of two distinct resolutions

# Consistency approach to resolution problem

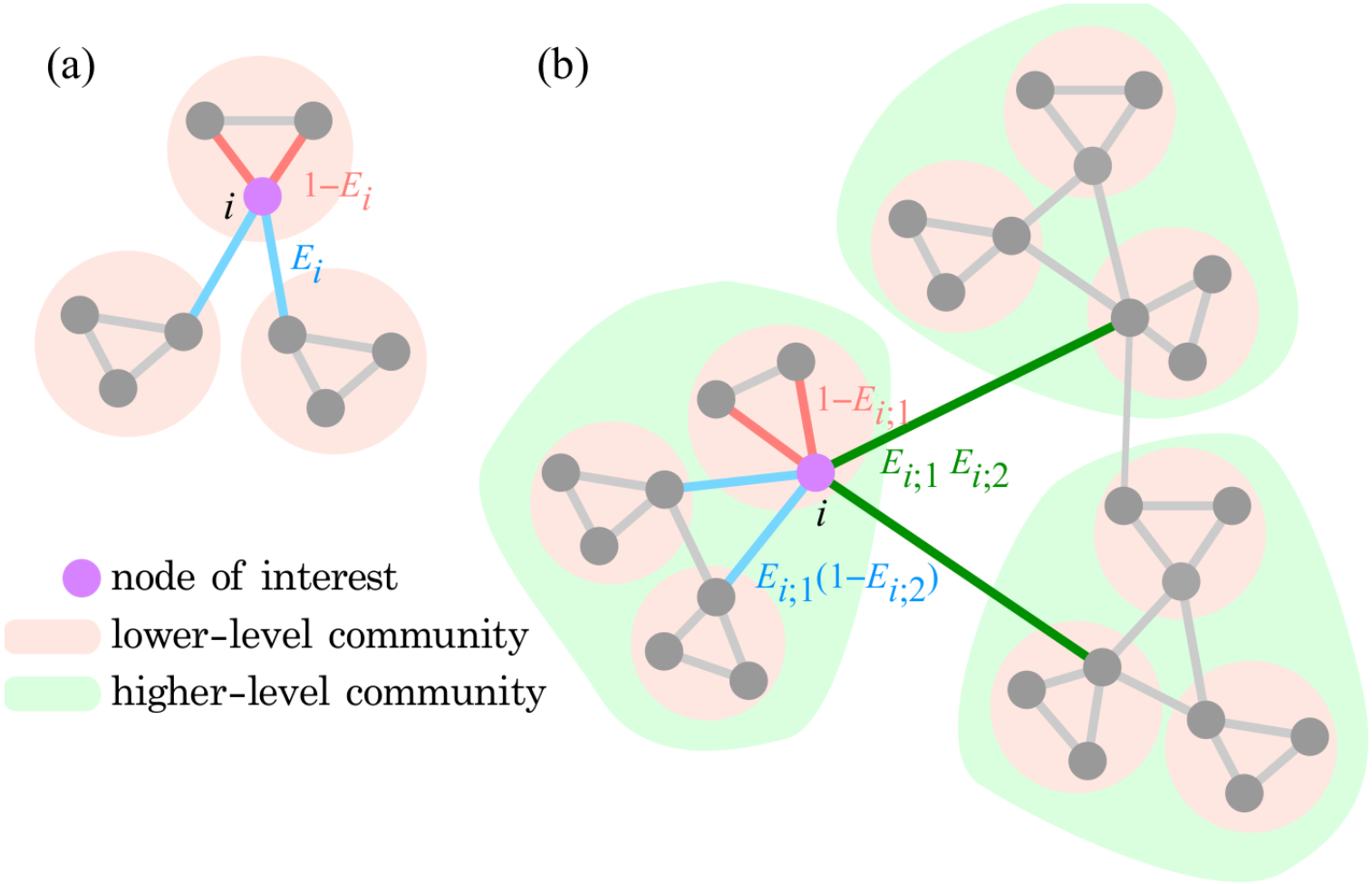





- Resolution of community structure can be determined by Partition Inconsistency
- Comparison of two distinct resolutions

# Result: model network



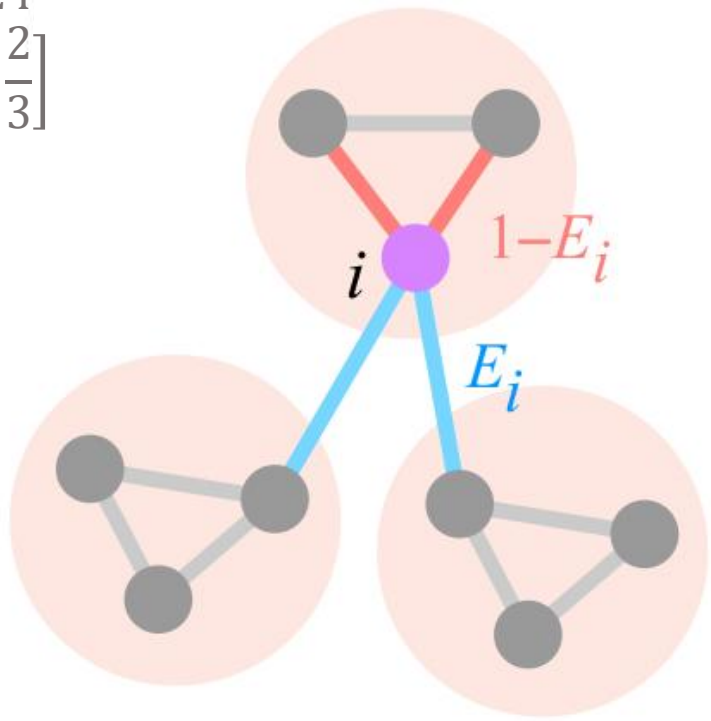


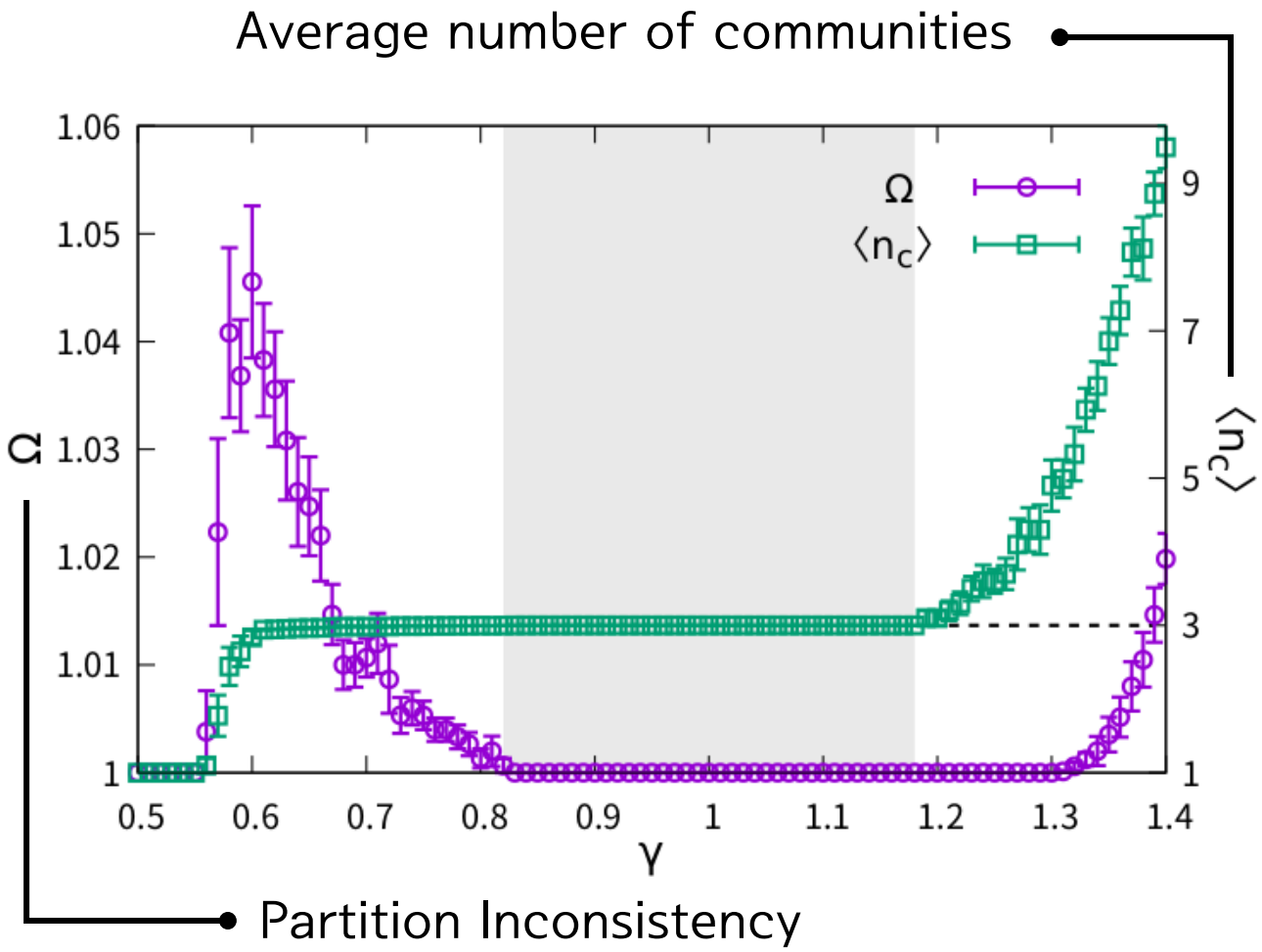

- Synthetic formation of benchmark model network
- Hierarchical community structure

# Result: model network



$n = 1200$
$p = 0.24$
$E_i = \left[0, \frac{2}{3}\right]$



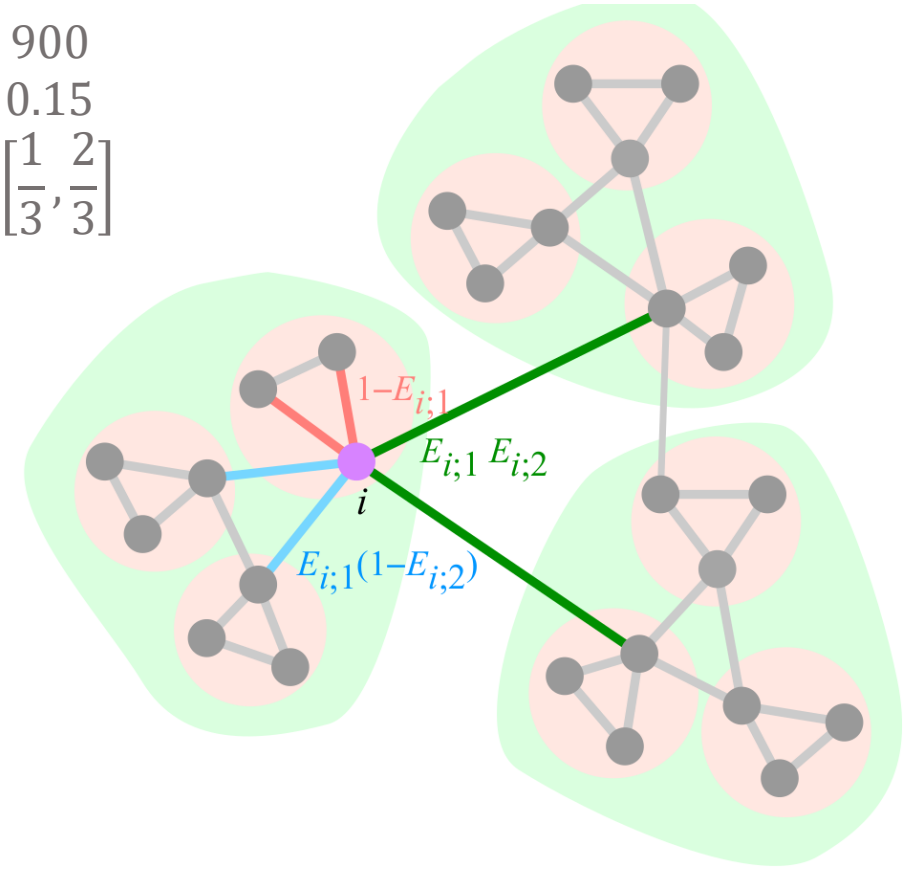



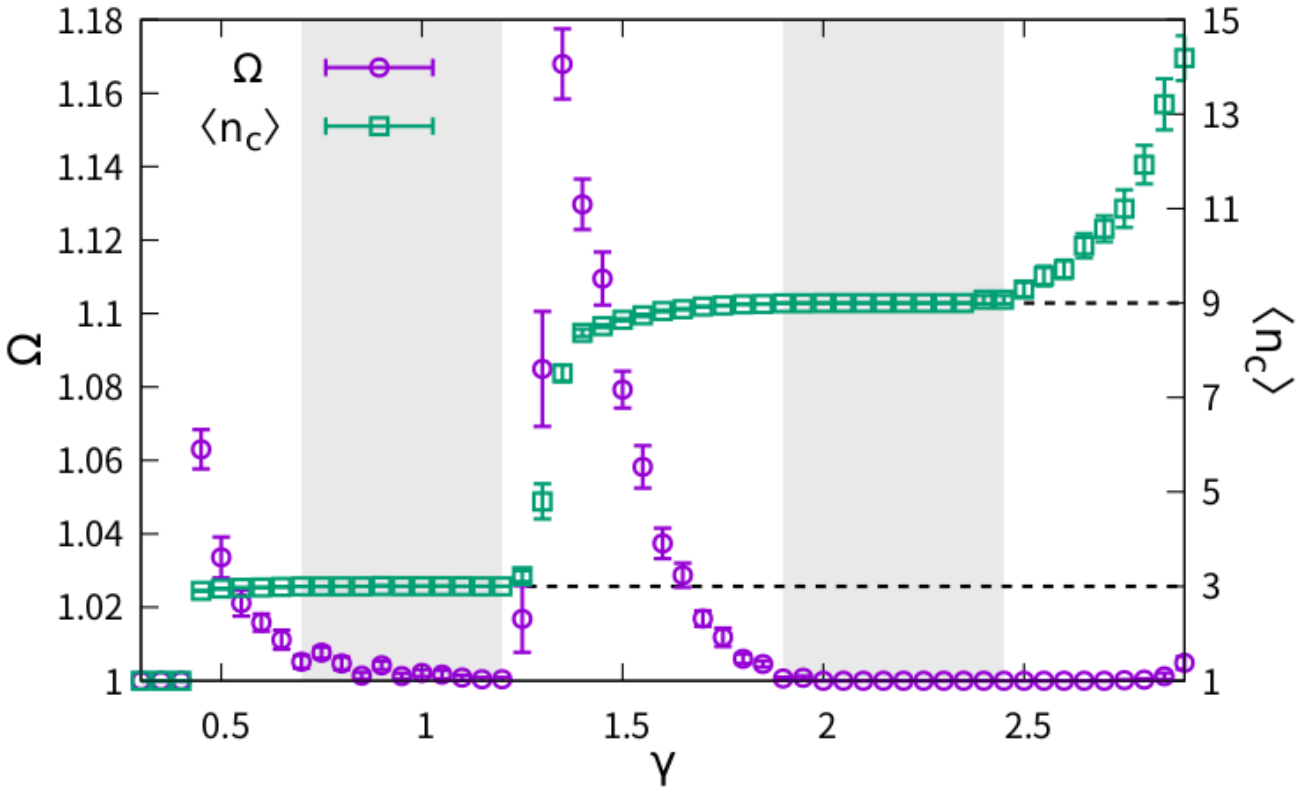

- Result of single-level network
- Low consistency region corresponds to the ground truth of model network

# Result: model network



$$n = 900$$
$$p = 0.15$$
$$E_i = \left[\frac{1}{3}, \frac{2}{3}\right]$$

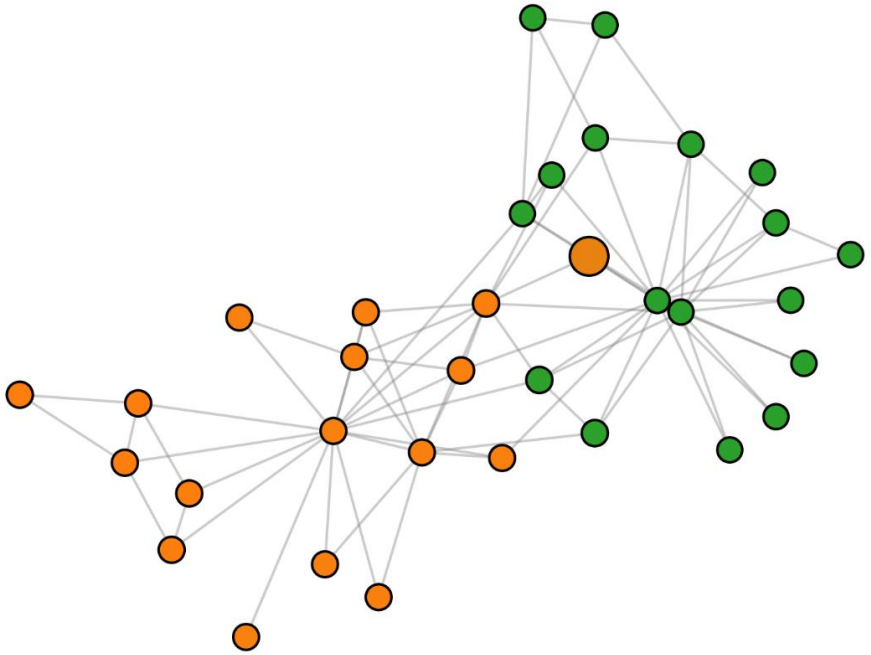


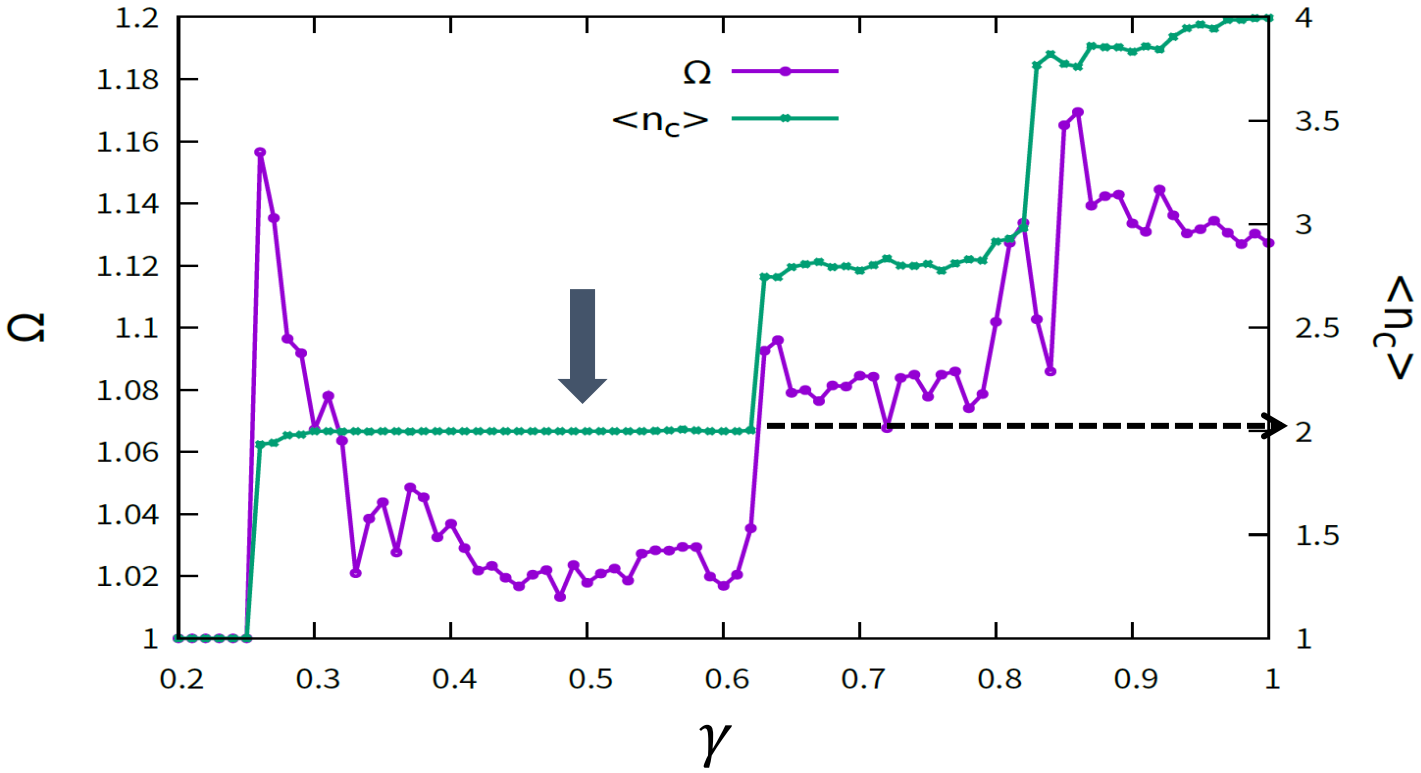




- Result of double-level network
- Low consistency region reconstruct both levels

# Data analysis: karate club network

Result

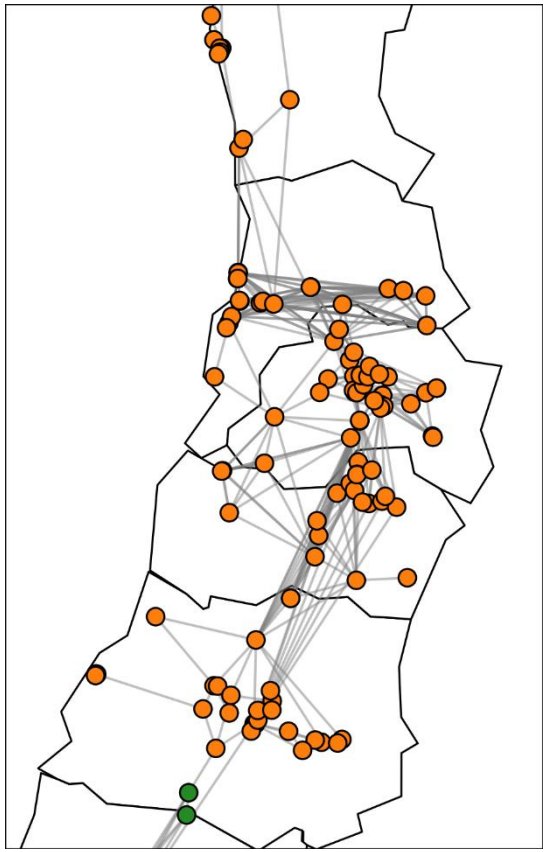


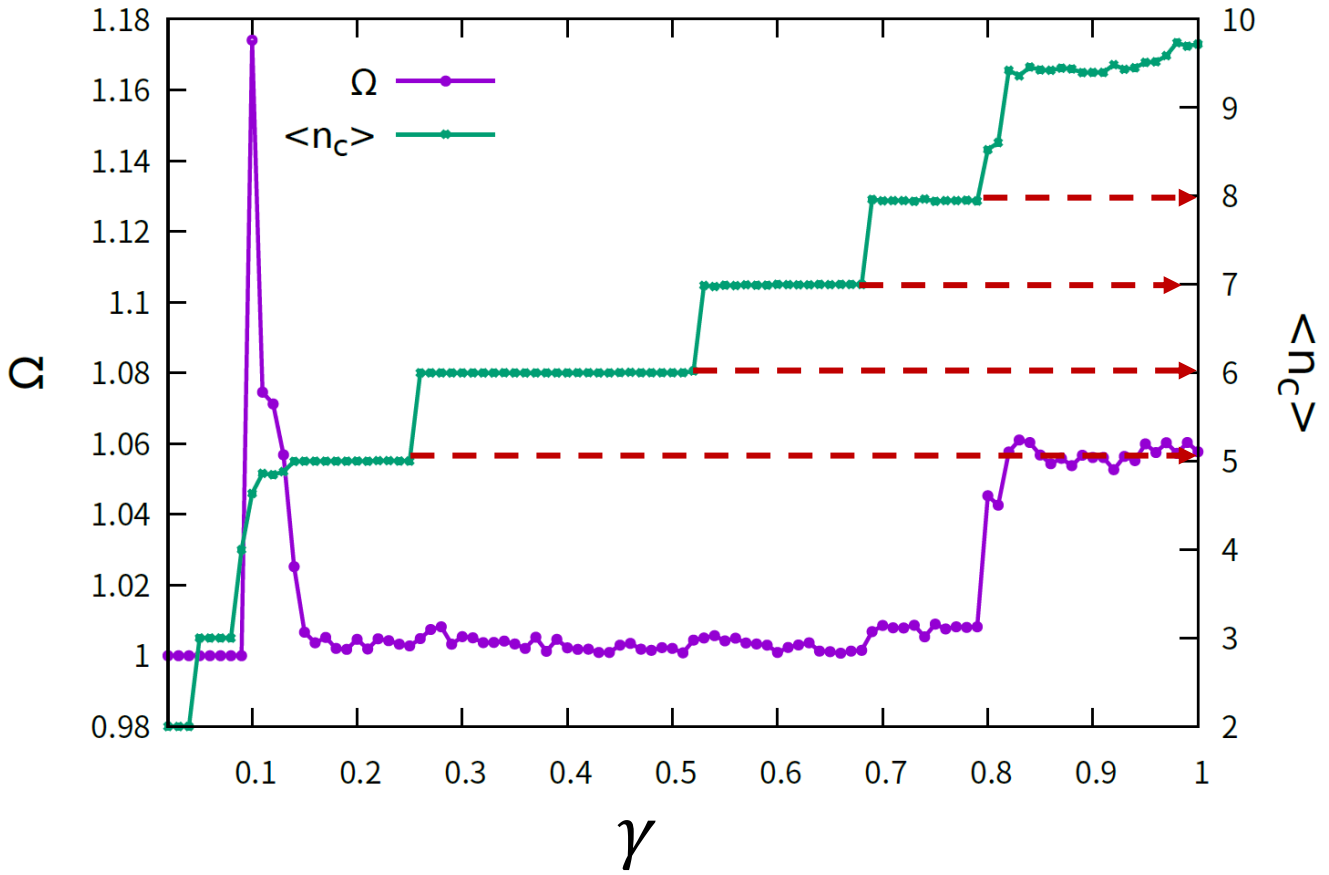



- Karate club network: A conventional benchmark of community structure
- Shows the lowest $\Omega$ when $n_c = 2$ → well matches with a prior knowledge

# Data analysis: Chilean power grid network

Result



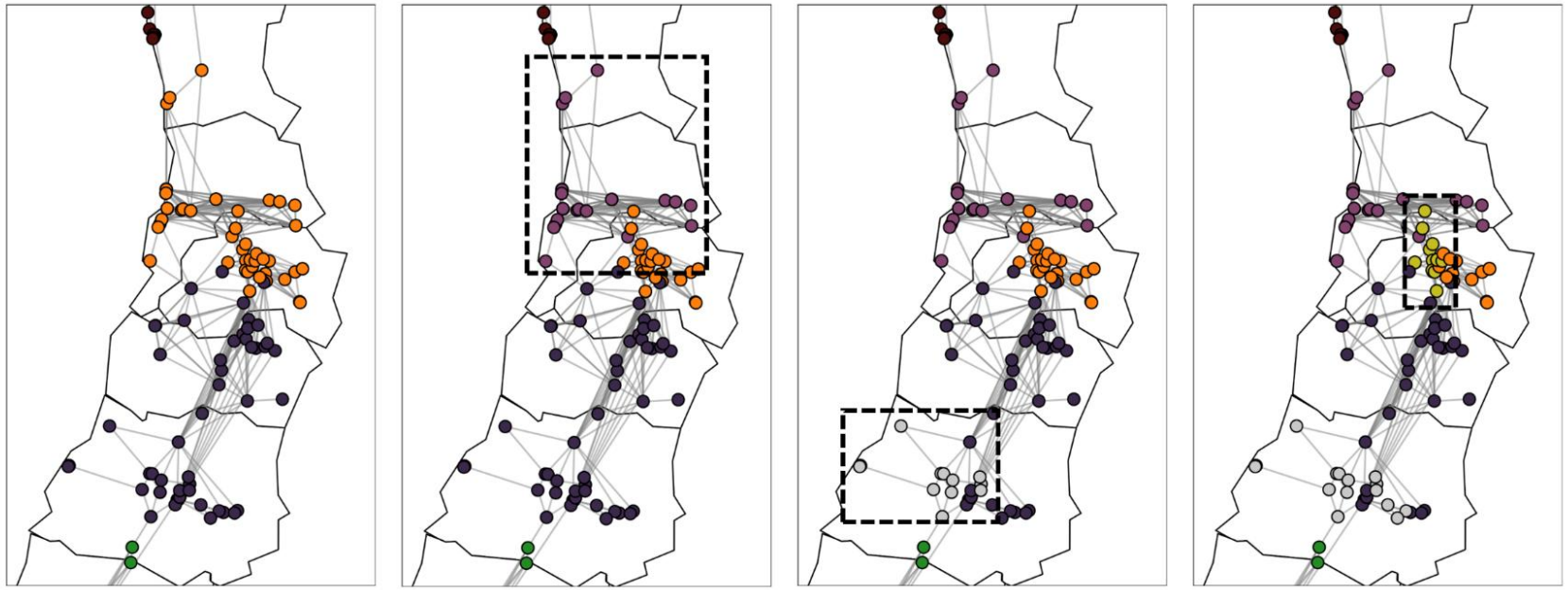



- A power grid network in Chile
- Shows low Ω in a wide range of γ → degeneracy of community structure?

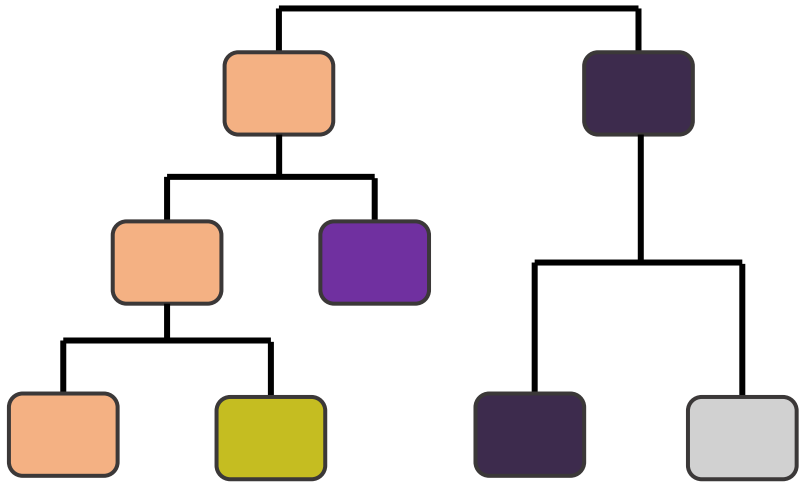
# Data analysis: Chilean power grid network

Result

$(a)\ \gamma = 0.2$ $(b)\ \gamma = 0.4$ $(c)\ \gamma = 0.6$ $(d)\ \gamma = 0.75$



Community structure



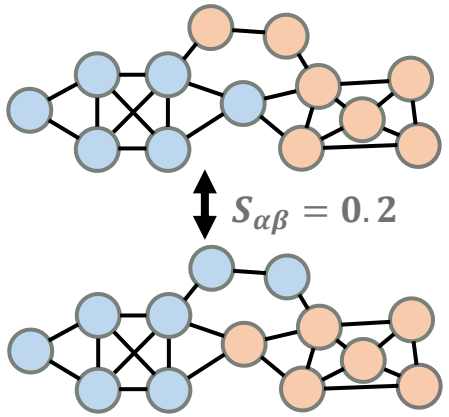
- Hierarchical community structure
- Smaller community structure emerges from larger communities

# Summary: Systematic analysis on community consistency



- Partition Inconsistency

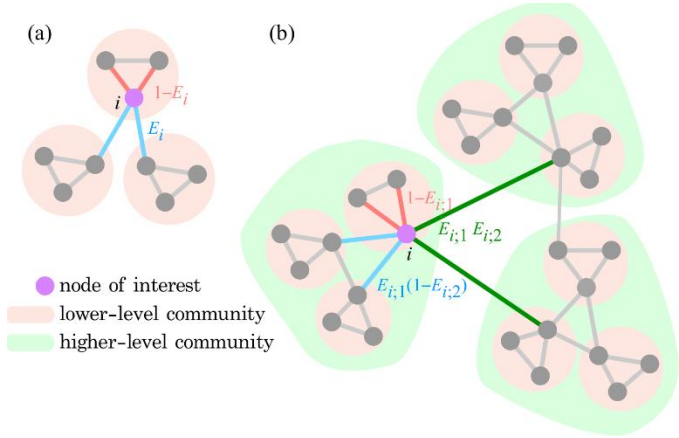
$P_\alpha = 0.6$

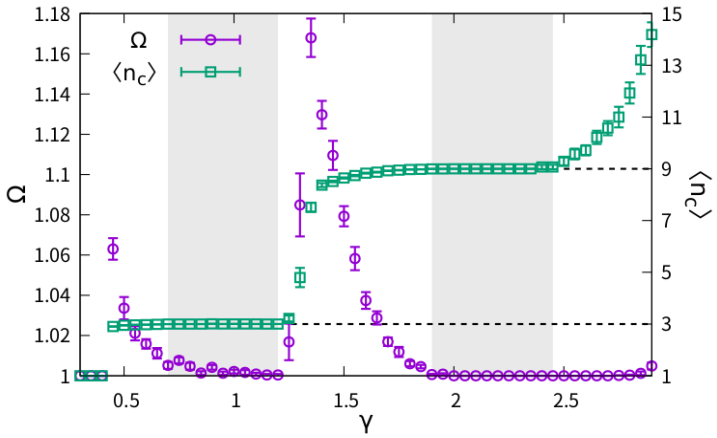



$P_\beta = 0.4$

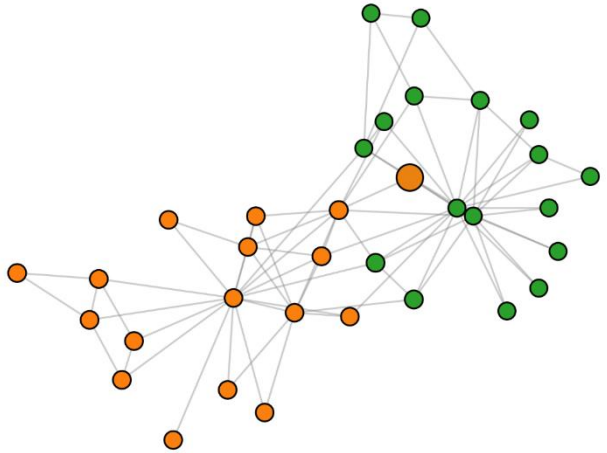
$$\Omega = \frac{1}{\langle S_{\alpha\beta} \rangle} = \left[ \sum_\alpha \sum_\beta P(C_\alpha) P(C_\beta) S_{\alpha\beta} \right]^{-1}$$

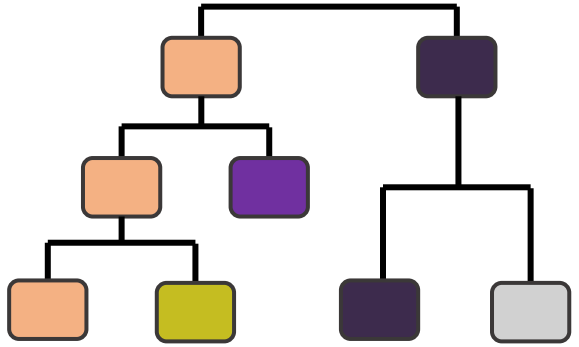
- Synthetic network




- Real network





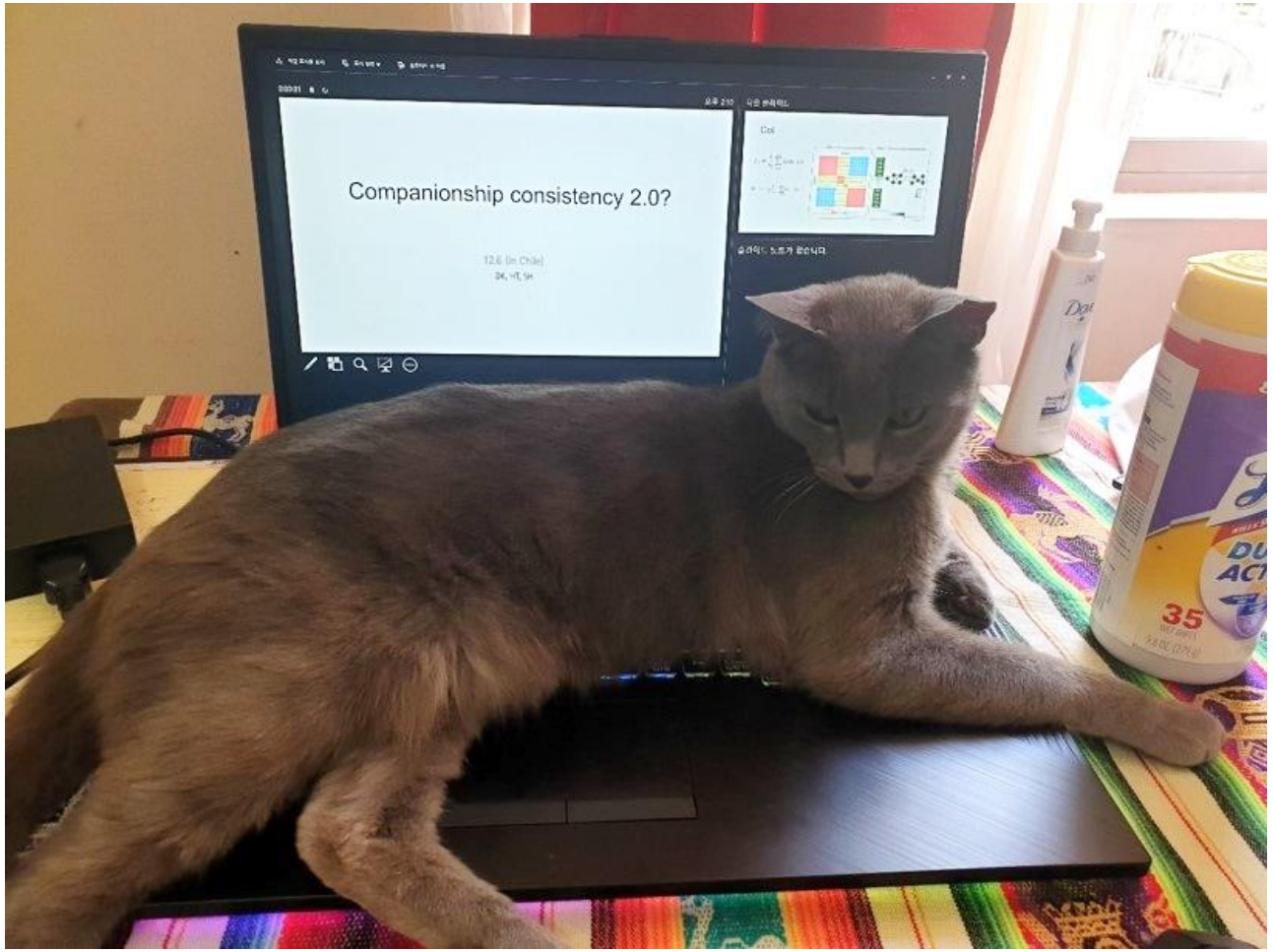
# Summary

-Introduced PaI to measure stability of community detection.

-Introduced MeI for node-level stability.

-Used both to analyze community structures across resolutions.

-Enables identifying optimal resolution and hierarchical organization.




1) Cat interrupting research (In Chile)

*Thanks for your attention*!